Capital, Intangibles, and Financial Frictions*

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Abstract

How do financial frictions shape firms' investment in intangible capital? We show that financial constraints distort firms' input choices, leading to systematic underinvestment in intangible assets. Exploiting an investment subsidy in Portugal that lowered the cost of both physical and intangible capital while keeping their relative price unchanged, we find that treated firms reduced their capital-to-intangible ratio by 12 percent, with larger effects for financially constrained firms. The distribution of treatment effects declines sharply across percentiles - firms with higher initial capital-to-intangible ratios adjust the most - revealing the signature of a binding financial wedge. Going beyond average effects, we recover the entire cross-sectional distribution of wedges between the marginal rate of technical substitution and the price ratio. The recovered distribution corresponds to the least distorted economy consistent with the data, showing that only a small share of firms are unconstrained while roughly one-quarter face wedges exceeding twenty percent - providing a direct quantitative map of financial distortions in production.

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1 Introduction

Intangible capital has become a central input in modern production. In the United States, the ratio of intangible to physical capital rose from 0.75 in 1975 to nearly one by 2021 (Crouzet et al., 2022). This shift has been closely associated with rising market concentration (Crouzet and Eberly, 2019) and increased market power (De Ridder, 2024), yet the adoption of intangible assets remains highly uneven across firms. A key reason is that intangible capital differs fundamentally from physical capital in its financing properties. Limited excludability makes it difficult to enforce property rights over the surplus it generates, rendering intangible assets poor collateral. As a result, investment in intangibles is predominantly financed with internal cash rather than external debt (Falato et al., 2022; Sun and Xiaolan, 2019). When returns to intangible capital are high, these financing frictions can lead firms—especially smaller and less liquid ones—to underinvest, widening productivity gaps and reinforcing concentration.

This paper studies how financial frictions distort firms' investment in intangible capital. We exploit the introduction of an investment tax subsidy in Portugal in 2014 that allowed firms to deduct a share of their investment expenditures—both physical and intangible—from their tax liability. Portugal provides a particularly suitable setting for studying financial frictions and intangible investment: the corporate sector is dominated by small and medium-sized firms, internal finance plays a central role, and intangible assets are predominantly adopted rather than internally produced. The policy lowered the cost of both inputs while leaving their relative price unchanged and, crucially, increased firms' internal liquidity. Its design generates quasi-experimental variation along two dimensions. First, eligibility was restricted to specific sectors, such as manufacturing, while excluding most services, agriculture, and mining. Second, the generosity of the subsidy varied across regions: firms in some areas could deduct up to 25 percent of investment costs, while others were limited to 10 percent.

To guide the empirical analysis, we develop a simple model in which firms allocate resources between physical and intangible capital in the presence of financial frictions. The key friction is that physical capital can be pledged as collateral, whereas intangible capital is poorly collateralizable. Firms with limited internal funds therefore face a higher effective cost of investing in intangibles and choose levels of intangible capital that are inefficiently low. In a frictionless environment, firms equate the marginal rate of technical substitution between physical and intangible capital to their relative price. Financial frictions break this condition by introducing a wedge that distorts input choices toward physical capital. Financially constrained firms thus operate with a capital-to-intangible ratio that is inefficiently high relative to the unconstrained benchmark. Importantly, because this wedge is defined in terms of relative input use within the firm, it can be identified without imposing assumptions on cross-firm differences in productivity, technologies, or price levels.

The model delivers three testable implications that discipline our empirical strategy. First, although the investment subsidy does not alter relative prices, it relaxes financial constraints by increasing internal liquidity. As a result, the wedge shrinks and treated firms reduce their capital-to-intangible ratio. Second, this adjustment should be concentrated among firms that were financially constrained prior to the introduction of the subsidy; firms operating close to the unconstrained optimum should exhibit little or no response. Third, the magnitude of the adjustment should be increasing in the firm's initial capital-to-intangible ratio: firms that start further from the unconstrained input mix—those with larger wedges—should experience larger reductions when constraints are relaxed. These implications provide a tight link between theory and data and imply that information contained in the cross-sectional pattern of responses can be used to learn about the underlying distribution of wedges in the economy.

We test these predictions using administrative data on Portuguese firms from 2010 to 2019, which provide comprehensive information on balance sheets, sectoral classification, and investment in both physical and intangible capital. We estimate the causal impact of the subsidy using a difference-in-differences framework that compares the evolution of the physical-to-intangible capital ratio among treated and control firms. Between 2014 and 2019, treated firms reduce this ratio by approximately 11 percent relative to control firms.

This decline is inconsistent with the predictions of a frictionless model but aligns closely with a framework featuring financial frictions. By increasing firms' cash holdings, the investment subsidy relaxes financing constraints and reduces the wedge between the marginal rate of technical substitution and the relative price of inputs, thereby shifting the capital mix even without altering relative prices. The observed adjustment in firms' input composition thus provides direct evidence that financial frictions shape investment decisions. Mapping these findings into our theoretical framework, we infer that the subsidy reduced the wedge by roughly 11 percent, implying that, prior to the policy, the average firm's capital-to-intangible ratio was at least 11 percent above its frictionless benchmark.

A key identifying assumption is that treated and control firms face similar growth rates in the relative price of physical and intangible capital. We assess this assumption by exploiting regional heterogeneity in the generosity of the subsidy. Because capital markets are national rather than regional, firms across regions should experience similar price trends. Consistent with this, treated and control firms in regions with smaller subsidies exhibit similar post-2014 investment patterns. In contrast, in regions where the subsidy was more generous, treated firms reduce their capital-to-intangible ratio by 17 percent relative to control firms. This pattern strengthens the interpretation that the results reflect a liquidity-driven relaxation of financial constraints rather than price effects.

Guided by the model, we next test whether the response to the subsidy is concentrated among financially constrained firms. We proxy for financial constraints using four alternative measures: total assets, the stock of physical capital, leverage, and the ratio of cash holdings to total assets. Across all measures, the evidence is consistent. Firms that were financially constrained prior to the subsidy exhibit a large and statistically significant decline in the capital-to-intangible ratio, while firms that were ex ante unconstrained display no economically meaningful response. For example, treated firms with low leverage show no change relative to controls, whereas treated firms with high leverage reduce their capital-to-intangible ratio by approximately 14 percent. This pattern rules out explanations based on changes in relative prices, accounting practices, or technology and confirms that the reallocation toward intangible capital is driven by the relaxation of binding financial constraints.

We also examine the extensive margin of adjustment. Treated firms are no more likely to adjust their stock of physical capital, but they are about 3 percent more likely to adjust their stock of intangible capital. This pattern indicates that the response operates primarily through the intensive margin, with some firms not adjusting simply because they are already at the unconstrained optimum rather than because of non-convex adjustment costs.

The investment subsidy further affects firms' balance sheets and performance. Treated firms increase cash holdings by approximately 4 percent relative to control firms and reallocate toward longer-term financing, reducing short-term debt by 7 percent while increasing long-term debt by 6 percent. Firm performance improves as well: treated firms experience a 3 percent increase in sales and a 4 percent increase in operating profits, with no corresponding change in operating margins. These results indicate real efficiency gains rather than changes in markups or pricing behavior.

Finally, we move beyond average effects and study the distributional implications of the subsidy. Using firms' heterogeneous responses, we recover quantile treatment effects across the initial distribution of the capital-to-intangible ratio. The pattern is sharply decreasing: firms near the bottom of the distribution—already close to the frictionless input mix—exhibit no response, while firms in the upper tail experience large declines. This downward-sloping profile is precisely what a model with financial frictions predicts and implies a compression in the distribution of wedges.

Building on this evidence, we develop a method to recover a lower bound on the cross-sectional distribution of wedges in the economy. Using the estimated quantile treatment effects, we construct the distribution of wedges that rationalizes the data with the least amount of distortion. We do not claim to recover the true distribution of wedges. Instead, we recover the allocation with the *least amount of distortions* consistent with both the theory and the observed firm-level responses.

The recovered distribution features a discrete mass point at one, corresponding to unconstrained firms. Approximately five percent of firms are located at this point, implying that at most five percent of firms operate without financial frictions. The median wedge is 1.13, almost identical to the lower bound for the average wedge inferred from the event study (1.12). Moreover, distortions are quantitatively significant: twenty-five percent of firms exhibit wedges of at least 1.2, meaning that their capital-to-intangible ratios are at least twenty percent above the frictionless benchmark.

Our recovered distribution is not the only one consistent with the data and the model. Any distribution that first-order stochastically dominates our recovered distribution would also satisfy the empirical and theoretical constraints. However, the distribution we estimate represents the allocation with the *least amount of distortions*—a natural lower bound on misallocation. In doing so, we provide a new way to measure the magnitude and shape of financial distortions in the economy.

Related Literature Our findings contribute to several strands of research on financial frictions, fiscal policy, and intangible investment.

A large literature studies how financial frictions distort resource allocation across firms and reduce aggregate productivity. Seminal contributions include Hsieh and Klenow (2009), Buera, Kaboski and Shin (2011), Midrigan and Xu (2014), Moll (2014), Greenwood et al. (2010), Bau and Matray (2023), and Ottonello and Winberry (2020). In macroeconomic theory, models such as Kiyotaki and Moore (1997) emphasize how credit constraints can amplify and propagate shocks through firms' investment decisions. We build on this insight by showing that financial frictions not only misallocate resources across firms but also distort *within-firm* investment decisions—causing firms to overinvest in physical capital relative to intangible capital, even when the latter offers higher returns. Our analysis highlights that misallocation operates both across firms and across asset types within firms.

Our paper is also related to work examining how fiscal policy affects investment behavior. Following the seminal contributions of Hall and Jorgenson (1967), Caballero, Engel and Haltiwanger (1995), and Cummins, Hassett and Hubbard (1994), a large empirical literature has shown that tax incentives can stimulate investment and output. Zwick and Mahon (2017) exploit variation in bonus depreciation and find that firms increase investment in response, with effects concentrated among smaller and financially constrained firms. Conversely, Yagan (2015) finds little evidence that dividend tax cuts affect investment, highlighting the importance of policy design. Our setting is closer to an accelerated depreciation scheme: the subsidy reduces the user cost of both tangible and intangible capital. However, because firms choose between two types of capital, the policy also affects the *composition* of investment, not just its level.

A third related literature studies the rise of intangible investment and its financing constraints. Gutiérrez and Philippon (2017) document a decline in aggregate investment, while Crouzet and Eberly (2023) show that this pattern largely reflects the growing importance of intangible assets. As Crouzet et al. (2022) and Eisfeldt and Papanikolaou (2013) emphasize, intangible capital differs from physical capital because it is difficult to collateralize and finance externally. Empirically, Falato et al. (2022) and Sun and Xiaolan (2019) show that intangible investment is predominantly funded with internal cash flows. We contribute to this literature by providing causal evidence that financial frictions create a wedge that leads firms to underinvest in intangible capital, and by showing how a liquidity shock—such as a tax subsidy—can relax this constraint.

The paper most closely related to ours is Ottonello and Winberry (2023), who study how financial constraints shape firms' trade-off between innovation and expansion. While their focus is on the aggregate implications of this trade-off, we examine smaller firms that invest in intangibles primarily as a form of *technology adoption*. Our findings highlight a complementary mechanism: rather than reallocating effort between innovation and scale, firms adjust the composition of their capital stock by substituting toward intangible assets when financing constraints are relaxed.

Outline. Section 2 explains the shock and describes the data. Section 3 describes the theoretical framework and the empirical results. Section 4 explains how we recover the distribution of wedges. Section 5 concludes.

2 The Portuguese Investment Subsidy

2.1 Description of the Policy Shock

In October 2014, Portugal introduced the *Regime Fiscal de Apoio ao Investimento*, a policy aimed at stimulating corporate investment and employment.[1] The program allows firms to deduct part of their investment from taxable income. Eligible expenditures include both tangible fixed assets and intangible capital.

The subsidy was not universal. Eligibility was restricted to a limited set of sectors, including mining, manufacturing, hospitality and accommodation, and technical services such as IT and R&D.[2] Firms outside these sectors form the control group, as they were not eligible for the subsidy.

The generosity of the subsidy varied across regions. Firms investing in Lisbon or the Algarve could deduct up to 10 percent of investment costs, while firms elsewhere could deduct up to 25 percent. In the analysis, we proxy investment location by the firm's registered address.[3] The sectoral eligibility cutoff allows comparisons between treated and untreated firms before and after the policy, while the regional variation in generosity provides an additional source of heterogeneity that we exploit in the analysis.

Firms are not required to claim the deduction in the year the investment occurs. If a firm reports negative taxable income or if its tax liability is smaller than the available deduction, the unused amount can be carried forward for up to two years. This provision is relevant, as roughly one third of Portuguese firms report negative net income.

The policy lowers the cost of both tangible and intangible capital without affecting their relative price, since it applies equally to both forms of investment. It also generates a direct shock to firms' net worth: holding investment policy fixed, end-of-period net worth increases mechanically. This mechanism is illustrated in Table I.

Table I considers a firm with constant sales and operating costs and reports free cash flow at the end of the period—that is, the cash generated by operations. In year 0, the firm invests 20 in either physical or intangible capital, fully financing the expenditure upfront. This investment increases depreciation in subsequent years.[4]

[1] Law-Decree 162/2014, dated October 31, established the subsidy. We treat 2015 as the first full year of implementation. The regime combined a corporate income tax deduction with exemptions from property taxes (for up to ten years), the real estate transfer tax, and stamp duty.

[2] Online Appendix Table A.I lists all eligible sectors. Online Appendix Figure A.1 reports the share of treated firms across employment, wages, sales, capital stock, and firm count. Treated firms account for roughly one quarter of all firms but represent about one third of total wages, sales, and capital stock.

[3] Most Portuguese firms operate a single establishment. Among multi-establishment firms, the majority concentrate operations within a single region.

[4] Depreciation schedules follow standard tax rules and are unaffected by the subsidy.

Without the subsidy, the only tax benefit from investing arises from future depreciation, which gradually reduces taxable income. With the subsidy, however, the firm can immediately deduct 5 from taxable income in year 0, increasing available cash by the same amount. The resulting rise in cash flow raises firm value one-for-one. As a result, both available cash and net worth increase in response to the subsidy.

In sum, the *Regime Fiscal de Apoio ao Investimento* creates a policy shock that is neutral in relative prices but expansionary in liquidity, effectively relaxing firms' financing constraints. Regional variation in generosity further provides cross-sectional heterogeneity that sharpens identification in the empirical analysis.

2.2 Data Sources

Our primary data source is the *Informação Empresarial Simplificada* (IES), a comprehensive administrative dataset covering the universe of Portuguese firms from 2010 to 2019. The IES is a joint initiative of the Ministry of Justice, Statistics Portugal, and the Bank of Portugal. The data are subject to formal consistency checks by the tax authorities and Statistics Portugal, ensuring comparability across firms and years. All incorporated firms are legally required to file standardized balance sheet and income statement information annually, providing near-complete coverage of the formal corporate sector.

The dataset provides detailed firm-level information, including industry classification, geographic location, and disaggregated measures of both the stock and flow of investment in tangible and intangible capital. These data allow us to construct consistent measures of physical and intangible investment, cash flow, leverage, and profitability, which are central to the empirical analysis that follows. We exclude sole proprietorships, which are not required to report PP&E and intangible assets separately. We also drop firms with non-positive values for sales, total assets, intangible assets, or PP&E.

Table II reports summary statistics for firms in both treated and control sectors. The dataset includes approximately 300,000 firms per year, yielding 2.7 million firm-year observations. The average firm is observed for 5.5 years, reflecting moderate attrition in the panel.[5]

In 2014, the year before the introduction of the investment subsidy, 24 percent of firms operated in treated sectors. Treated firms tend to be larger: their average sales are 38 percent higher than those of control firms, and they employ roughly twice as many workers. By contrast, leverage, productivity (measured as EBITDA-to-sales), and cash holdings do

[5]Most of the analysis relies on this unbalanced panel. Restricting the sample to a balanced panel yields similar results.

not differ significantly between groups. Treated firms also hold more physical capital but report a smaller stock of intangible assets relative to control firms.

We measure physical and intangible capital using the book values reported in firms' balance sheets. There are two potential concerns with this approach.

First, internally developed intangible assets are recorded at cost rather than at market value. For example, if a firm develops a patent, the book value reflects the expenditures incurred during development rather than the patent's expected market value (Kogan et al., 2017). When firms can accurately forecast both the value and cost of such assets, book values will systematically understate their economic value. This issue applies broadly to a wide class of internally produced intangibles.[6] In our context, however, most firms—large or small—are unlikely to be producers of intangible capital; instead, they are adopters who purchase intangible assets such as software or licenses. In such cases, the book value coincides with the market value at the time of acquisition, reducing the concern about systematic undervaluation.

Second, except in the year the asset is purchased, book values may diverge from market values due to depreciation schedules, amortization rules, or other accounting conventions. Provided that these discrepancies are not systematically correlated with treatment, they do not bias our estimates. Any firm-specific component of this measurement error is absorbed by firm fixed effects, while time-specific components are absorbed by year fixed effects. We return to this issue in Section 3, where we show that our ratio-based analysis alleviates many of these concerns.

A further advantage of our empirical strategy is that focusing on the *ratio* of physical to intangible capital makes our estimates substantially more robust to measurement error than analyses based on levels alone. If book values differ from market values by multiplicative, asset-specific factors—arising, for example, from amortization rates or valuation conventions—these factors may cancel out when taking ratios. Moreover, any measurement error that is constant within firms or common across time is absorbed by the corresponding fixed effects. As a result, our use of the capital-to-intangible ratio provides a cleaner measure of firms' input composition and a more reliable basis for assessing the effects of financial frictions.

We also measure physical and intangible capital using their book values rather than physical quantities. This choice reflects a data limitation: we do not observe transaction-

[6] A common solution in the literature is to recover the value of intangible assets recursively. For example, Eisfeldt and Papanikolaou (2013) and Crouzet et al. (2022) define intangible capital via $\text{Int}_{t+1} = (1-\delta)\text{Int}_t + x_t$, where x_t is intangible investment typically proxied by SG&A. Given an initial value Int_0, one can construct a time series for intangible capital. However, our panel has a short time dimension, which makes this approach unreliable.

level prices at which firms acquire individual assets, which would be necessary to recover true quantities. Deflating book values using sector-level price indices may seem like a natural alternative, but this approach does not reliably identify quantities.[7] For this reason, and because our empirical strategy relies on changes in relative capital inputs rather than levels of physical quantities, we rely on book values throughout the analysis.

Our goal is to understand how the investment subsidy affects the distribution of the capital-to-intangible ratio. Figure 1 plots the distribution of the logarithm of this ratio for all firms in 2014. On average, treated firms hold more physical capital relative to intangible assets, and most of the difference arises from the left tail of the distribution. The variance of the logarithm of the capital-to-intangible ratio is large for both groups: for treated firms, it equals 92 percent of the cross-sectional mean, and for control firms, 93 percent.

To assess how these differences may interact with wedges, we first document how much of the observed heterogeneity arises within versus across sectors. The large variance in the capital-to-intangible ratio documented in Figure 1 motivates a simple decomposition of this variation. Although we formally introduce the theoretical structure in the next section, here we provide an empirical preview of the underlying heterogeneity. The variance may reflect differences in production structure, relative prices, or wedges between the marginal rate of technical substitution and relative prices. Using data through 2014, we regress the logarithm of the capital-to-intangible ratio on a series of fixed effects and report the resulting R^2 values in Table III.

Column (1) shows that differences across sectors—defined at the four-digit industry level—explain only 16 percent of the variation in the capital-to-intangible ratio. Allowing for time-varying sectoral effects, as in column (2), does not raise the explanatory power. If cross-sector differences in production functions were the main source of variation, we would expect a larger share of the variance to be explained given our narrow industry classification. In contrast, firm-level differences within sectors, shown in column (3), account for most of the variation in the capital-to-intangible ratio. This pattern

[7] To see the problem, let the book value of an intangible asset purchased at time 0 and observed at time t be $p^h_{i,0}\, h_{i,t}$, where $p^h_{i,0}$ is the firm-specific purchase price. Let $\bar{p}^h_t$ be a sector-level price index. Deflating yields

$$\hat{h}_{i,t} = \frac{p^h_{i,0}}{\bar{p}^h_0} \cdot \frac{\bar{p}^h_0}{\bar{p}^h_t} \cdot h_{i,t},$$

so the estimated quantity differs from the true quantity for two reasons. First, the ratio $p^h_{i,0}/\bar{p}^h_0$ captures within-sector price dispersion at the time of purchase, introducing firm-year–level measurement error. Second, the ratio $\bar{p}^h_0/\bar{p}^h_t$ captures the evolution of the (sector-wide) price index, introducing sector-year–level measurement error. Neither source of error can be eliminated without observing the firm-level prices at acquisition. As a result, sector-level deflation cannot recover true quantities.

suggests—without yet imposing a structural interpretation—that differences in relative prices or financial wedges are the primary sources of heterogeneity in the data.

2.3 Aggregate Effects

We begin by examining aggregate patterns in investment composition before turning to firm-level outcomes. The analysis focuses on the effect of the investment subsidy on the ratio of physical to intangible capital. Figure 2 plots the aggregate stocks of physical and intangible capital for treated and control firms over time. The series move in parallel before the introduction of the subsidy and diverge thereafter, showing a clear break in trend following the policy.

The subsidy led to increases in both physical and intangible capital at the aggregate level, with treated firms experiencing larger gains than control firms. The expansion in intangible capital, however, was substantially stronger. Between 2014 and 2019, treated firms increased their stock of physical capital by 16 percent more than control firms, while the corresponding increase in intangible capital was 38 percent.[8] As a result, the ratio of physical to intangible capital declined by 22 percent, as shown in Online Appendix Figure A.3.

This decline is difficult to reconcile with a frictionless model of the firm, as the relative prices of physical and intangible capital remained unchanged. Instead, the shift in the input mix is consistent with the presence of financial frictions, which create a wedge between the marginal rate of technical substitution and relative input costs.

The observed changes reflect not only higher capital stocks but also greater investment activity. Online Appendix Figure A.4 shows that, relative to control firms, treated firms increased investment in physical capital by 4 percentage points and in intangible capital by 18 percentage points. Moreover, treated firms became more likely to invest in intangible capital at all: the probability of any intangible investment rose by 3 percentage points, as shown in Online Appendix Figure A.5.

Taken together, these aggregate patterns indicate that the subsidy expanded overall investment while shifting its composition toward intangibles—a reallocation consistent with a liquidity-driven response rather than a change in relative prices.

[8] The increase in intangible capital is not driven by goodwill. Online Appendix Figure A.6 shows that it is largely accounted for by software acquisitions and patent production.

3 The Effect on Firm-Level Investment

This section brings the theory to the data. We first lay out a minimal framework in which an investment subsidy affects the capital mix only through a wedge between the marginal rate of technical substitution and relative input prices. Guided by this structure, we estimate an event–study that traces the dynamics of the capital–to–intangible ratio. Next, we turn from composition to levels to assess whether the reallocation reflects substitution or expansion. We conclude with reduced–form evidence on firm performance, reporting how sales, employment, profitability.

3.1 A Simple Model

Before describing our empirical strategy, we outline a simple model of a firm that combines physical and intangible capital. A key feature of the environment is that firms are *adopters* rather than producers of intangible capital. This distinction matters because adopted intangibles—such as software, licenses, or organizational technologies—are difficult to pledge as collateral, giving rise to financing frictions.

The firm produces output using physical capital k and intangible capital h according to the CES production function

$$y = A\left[\alpha(\phi_k k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\phi_h h)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where A denotes total factor productivity, $\alpha \in (0,1)$ is the physical capital share, ϕ_k and ϕ_h are input-specific productivity terms, and $\sigma > 0$ is the elasticity of substitution between inputs. The firm is a price taker in all markets. The prices of physical and intangible capital are p_k and p_h, respectively, and the output price is normalized to one.

Investment in intangible capital is subject to a financial constraint. Physical capital can be freely chosen, but intangible capital must be financed out of internal resources. Specifically, we assume

$$p_h h \leq \lambda a, \tag{2}$$

where $a > 0$ is firm net worth and $\lambda > 0$ captures the degree to which intangible capital can be collateralized. A higher λ corresponds to weaker financial frictions. This formulation follows Moll (2014) and reflects the limited pledgability of intangible assets documented in Crouzet et al. (2022) and Ottonello and Winberry (2023).

The firm chooses (k, h) to maximize profits subject to (2). The solution is summarized in the following proposition.

Proposition 1 *The firm's optimal input choice satisfies*

$$\left(\frac{k}{h}\right)^{\frac{1}{\sigma}} = \frac{\alpha}{1-\alpha}\left(\frac{\phi_k}{\phi_h}\right)^{\frac{\sigma-1}{\sigma}} \frac{p_h}{p_k}(1+\mu), \tag{3}$$

where $\mu \geq 0$ is the Lagrange multiplier on the financial constraint (2). There exists a threshold $\tilde{a}$ such that $\mu(a) = 0$ for $a \geq \tilde{a}$ and $\mu(a) > 0$ for $a < \tilde{a}$. Moreover, for $a \in (0, \tilde{a})$, the multiplier $\mu(a)$ is weakly decreasing in net worth.

Proof. In Appendix B. ■

When the firm is unconstrained ($\mu = 0$), it equates the marginal rate of technical substitution to the relative price ratio, yielding the frictionless input mix. Financial frictions introduce a wedge $\omega \equiv 1 + \mu \geq 1$, which raises the physical-to-intangible capital ratio above its unconstrained level.

In the data, we observe capital stocks in values rather than quantities. Rewriting (3) in value terms and taking logs yields

$$\log\left(\frac{p_k k}{p_h h}\right) = \sigma \log\frac{\alpha}{1-\alpha} + (\sigma-1)\log\frac{\phi_k}{\phi_h} + (\sigma-1)\log\frac{p_h}{p_k} + \sigma\log\omega. \tag{4}$$

An increase in the wedge ω—that is, tighter financial constraints—raises the observed capital-to-intangible ratio in values.

The investment subsidy we study lowers the cost of both types of capital proportionally and therefore does not affect relative prices. Define the average treatment effect on changes in any variable x as

$$\text{ATT}(x) \equiv \mathbb{E}[d\log x \mid \text{Treated}] - \mathbb{E}[d\log x \mid \text{Control}].$$

Assume that treatment is orthogonal to changes in relative prices and relative input productivity,

$$\text{ATT}\left(\frac{p_k}{p_h}\right) = 0, \qquad \text{ATT}\left(\frac{\phi_k}{\phi_h}\right) = 0.$$

Under these conditions, equation (4) implies

$$\text{ATT}(\omega) = \frac{1}{\sigma}\text{ATT}\left(\frac{p_k k}{p_h h}\right). \tag{5}$$

Because the subsidy relaxes financing constraints only for treated firms, wedges for control firms are unchanged. Changes in the capital-to-intangible ratio therefore identify changes in financial wedges.

The model delivers three testable predictions that guide our empirical analysis.

1. *Average effect.* A liquidity shock that relaxes financial constraints should reduce the capital-to-intangible ratio among treated firms relative to controls, even though relative prices are unchanged.

2. *Firm-level heterogeneity.* The decline in the capital-to-intangible ratio should be larger for firms that are more financially constrained prior to treatment, as these firms face larger wedges.

3. *Distributional implications.* Conditional on common prices and technology, firms with a higher initial capital-to-intangible ratio—those further from the frictionless input mix—should exhibit larger declines following the subsidy.

These predictions allow us to distinguish financial frictions from alternative explanations based on prices or technology and motivate both our heterogeneity analysis and our distributional approach.

A related concern is whether adjustment costs, rather than financial frictions, could explain the observed changes in the capital–to–intangible ratio. Even in the presence of convex adjustment costs in a dynamic model, a frictionless firm's optimal input mix is still pinned down by relative prices. With quadratic adjustment costs, the first–order conditions imply that the marginal rate of technical substitution between physical and intangible capital equals the relative price ratio, up to transitory terms that govern the speed of adjustment but not the composition of inputs. As a result, if a policy leaves relative prices unchanged—as in our setting—a frictionless model predicts no systematic change in the capital mix. This conclusion continues to hold even when adjustment costs differ across capital types: such asymmetries affect the magnitude and timing of investment responses, but do not generate a wedge between the marginal rate of technical substitution and the price ratio. Adjustment costs therefore shape dynamics, but cannot by themselves produce the observed reallocation toward intangible capital. Only a relaxation of financial constraints can shift the capital mix in this setting.

Equation (5) shows that recovering the magnitude of wedge changes requires knowledge of the elasticity of substitution σ. In principle, one might attempt to estimate σ from equation (4). However, this is not feasible in our setting. The wedge ω is an omitted variable that depends on prices, so ordinary least squares would suffer from omitted-variable bias, and the sign of the bias cannot be determined because μ is generally a non-monotonic function of prices. Estimation in a subsample of unconstrained firms would require firm-level price data, which we do not observe.

We therefore adopt the conservative assumption $\sigma = 1$, corresponding to a Cobb–Douglas production function. Under this assumption, changes in the observed capital-to-intangible ratio map one-for-one into changes in the wedge. Allowing for $\sigma \neq 1$ would rescale the magnitude of wedge changes but would not affect any of the qualitative or distributional predictions of the model. Throughout the paper, we interpret our estimates as conservative measures of the extent to which financial frictions distort firms' input choices.

In the next subsection, we bring these predictions to the data and estimate the average and heterogeneous effects of the investment subsidy on firms' capital composition.

3.2 Effects on the Capital-to-Intangible Ratio

We estimate the average treatment effect of the investment subsidy on the ratio of physical to intangible capital using the following event–study specification:

$$\log\left(\frac{k_{i,t+1}}{a_{i,t+1}}\right) = \alpha_i + \lambda_t + \beta X_{i,t} \\ + \sum_{m=-5,\, m\neq -1}^{4} \delta_m \,\mathbf{1}\{i \in \text{Treated}\}\, \mathbf{1}\{t = 2015 + m\} + \varepsilon_{i,t}, \tag{6}$$

where the dependent variable is the logarithm of the ratio of the value physical capital $k_{i,t+1}$ to the value intangible capital $h_{i,t+1}$ for firm i at the end of year t. The specification includes firm fixed effects α_i, year fixed effects λ_t, and a vector of time–varying controls $X_{i,t}$.

The control vector $X_{i,t}$ includes the logarithm of sales, total assets, and employment; leverage; the ratio of cash holdings to total assets; the EBITDA–to–sales ratio; the share of physical capital in total assets; and the logarithm of total sectoral sales.

The coefficients of interest are the event–time indicators δ_m, which measure the differential change in the capital–to–intangible ratio of treated firms relative to control firms, using 2014 ($m = -1$) as the omitted reference year. Standard errors are clustered at the firm level.

Identification of the coefficients δ_m follows directly from the structure outlined above. Year fixed effects absorb aggregate movements in the relative price of physical and intangible capital, ensuring that the estimated δ_m coefficients capture changes in the wedge ω rather than shifts in p_k/p_h in Equation (4). Firm fixed effects control for time-invariant heterogeneity in production technology, preferences, and financing capacity, while time-varying controls account for shocks to productivity, demand, or firm-specific liquidity. In particular, we include controls for sales, leverage, and cash holdings, which help absorb exposure to heterogeneous financial shocks.

Our identification strategy assumes that, conditional on these observables, firms not eligible for the subsidy provide a valid counterfactual for treated firms. In practice, this means comparing treated firms in sectors such as manufacturing with control firms in sectors such as agriculture. While the assumption cannot be tested directly, it implies a falsifiable prediction: treated and control firms should display parallel pre-treatment trends in the capital-to-intangible ratio. We verify this condition in the event-study results below.

The results from estimating equation (6) are shown in Figure 3. We find no statistically significant differences in the capital-to-intangible ratio between treated and control firms before the introduction of the policy, supporting the validity of the parallel-trends assumption. Following the implementation of the investment subsidy, treated firms reduce their ratio of physical to intangible capital relative to control firms. Between 2014 and 2019, the average treated firm experiences an 11 percent decline in this ratio.

A natural concern is that the book values of physical or intangible capital may be measured with error. Let the observed stocks be

$$k_{i,t+1} = k^{\star}_{i,t+1} u^{k}_{i,t+1}, \qquad h_{i,t+1} = h^{\star}_{i,t+1} u^{h}_{i,t+1},$$

where $x^{\star}$ is the true value and u is measurement noise. Measurement error is problematic only if firms systematically change their reporting of assets *because of* the subsidy.[9] For example, suppose that

$$\log u^{j}_{i,t+1} = \theta^{j}\, \mathbf{1}\{t \geq 2014\} \mathbf{1}\{i \in \text{Treated}\}$$

In that case, the estimated coefficients would identify

$$\delta_m = \tau_m + (\theta^h - \theta^k), m \geq 0$$

where τ_m is the true treatment effect. If reporting changes affect physical and intangible capital symmetrically, the difference $\theta^h - \theta^k$ is zero and the estimates remain valid. If, instead, firms overstate intangible assets relative to physical ones after treatment, then δ_m is an upper bound on the true effect. Since our estimates are negative, this would imply that the true decline in the ratio is *larger* in magnitude.

[9]If the measurement-error component is either i.i.d. or driven by firm and year effects, then the fixed effects in equation (6) absorb it, and the event–study estimates remain consistent.

To assess whether firms adjusted their reporting practices, we examine selling, general, and administrative expenses (SG&A). A shift from owning to renting assets would lower book values while raising SG&A. In Online Appendix Figure C.1, we re-estimate equation (6) using SG&A scaled by sales, by the stock of intangible assets, and by the stock of physical assets. We find no differential post-treatment changes for treated firms relative to controls. This provides no evidence that firms altered their reporting of assets in ways correlated with treatment.

Under the assumption that relative input prices are unaffected by treatment status, this decline reflects a reduction in the distortion wedge ω. In a Cobb–Douglas setting (where $\sigma = 1$), this corresponds directly to an 11 percent decrease in the wedge. The results therefore indicate that the investment subsidy relaxed financing constraints, consistent with the view that financial frictions distort firms' capital allocations.

The findings are robust to alternative explanations. Online Appendix Figure C.3 shows that the estimated effect is similar across firms with and without taxable profits, consistent with the fact that even unprofitable firms benefit from the subsidy as they may deduct the investment in future taxes. Online Appendix Figure C.2 confirms that the results also hold in a balanced panel, indicating that entry and exit dynamics are not driving the observed reallocation.

Following Bau and Matray (2023), the fact that the wedge is bounded below by one allows us to reinterpret the event–study estimate of the change in the capital–to–intangible ratio as a lower bound on the initial average wedge in the economy. Under a Cobb–Douglas production function, the change in the average capital–to–intangible ratio is identical to the change in the average wedge.[10] Because $\omega_i \geq 1$ for each firm, the initial wedge $\omega_{i,0}$ is bounded from below by e^{-d_i}, where d_i denotes the individual treatment effect in logs. Taking expectations, applying Jensen's inequality, and using the consistency of our average treatment effect estimates yields $\mathbb{E}[\omega_0] \geq e^{-d}$, where d is the estimated average treatment effect.

The estimated decline of $d = -0.11$ in the average capital–to–intangible ratio therefore implies that the average wedge in the economy prior to the subsidy was at least 1.12. In other words, the effective relative cost of intangible capital was at least 12 percent above its frictionless level. Within our framework, this also implies that the capital–to–intangible ratio itself was at least 12 percent above the frictionless benchmark — consistent with a sizable compositional distortion that the policy subsequently alleviated.

Taken together, these results show that the investment subsidy substantially reduced

[10] This equivalence holds even if capital shares are heterogeneous across firms, provided that these shares do not change systematically with treatment over time.

distortions in firms' input choices, narrowing the wedge that constrained intangible investment. In the next section, we examine how this adjustment varies across firms with different financial positions and balance-sheet characteristics.

3.3 Heterogeneous Effects

We next examine how the effects of the investment subsidy vary across regions and firms. Heterogeneity in the treatment response is informative along two dimensions. First, regional variation in the generosity of the subsidy allows us to test whether our results could be driven by changes in relative prices. Second, variation across firms provides a direct test of the mechanism: if the wedge ω reflects financial frictions, we should observe stronger effects among financially constrained firms.

We begin by exploiting the policy's regional variation in generosity. Firms located in Lisbon and the Algarve could deduct up to 10 percent of investment costs, whereas firms elsewhere could deduct up to 25 percent. We re-estimate equation (6) separately for high- and low-subsidy regions, and present the results in Figure 4.

Among firms in low-subsidy regions, we find no statistically significant change in capital mix. In contrast, treated firms in high-subsidy regions experience a 17 percent decline in the capital-to-intangible ratio relative to control firms between 2014 and 2019. This pattern suggests that small subsidies are insufficient to induce firms to adjust their capital composition, while larger subsidies meaningfully relax financial constraints. The result is consistent with models featuring fixed adjustment costs—such as Khan and Thomas (2008)—in which small shocks fail to overcome inaction thresholds.

Regional heterogeneity also helps rule out alternative explanations based on differential price movements. If the relative price of intangible capital had fallen more sharply for treated firms—perhaps because of sector-specific input demands—we would expect to see similar effects in all regions. Instead, we observe large effects only in high-subsidy areas. Given that capital markets are nationally integrated, this pattern is difficult to reconcile with price-based explanations and instead supports the interpretation that the subsidy operated through liquidity rather than prices.

We now examine how the effects of the investment subsidy vary across firms with different financial positions. The theoretical framework predicts that the subsidy should have the strongest impact on financially constrained firms, which face the largest wedges between the marginal rate of substitution and relative input prices. To test this prediction, we estimate the following triple–difference specification:

$$\log\left(\frac{k_{i,t+1}}{h_{i,t+1}}\right) = \alpha_i + \lambda_t + \beta X_{i,t} + \delta_0\,\mathbf{1}\{i \in \text{Treated}\}\mathbf{1}\{t \geq 2015\} + \delta_1\,\mathbf{1}\{i \in \text{Treated}\}\mathbf{1}\{t \geq 2015\}\mathbf{1}\{i \in \text{Fin. Constrained}\} + \varepsilon_{i,t}, \tag{7}$$

where the vector of controls $X_{i,t}$ includes the interaction between the post–treatment indicator and the financial–constraints indicator. This specification identifies the differential treatment effect for financially constrained firms relative to unconstrained firms. Consistent with the model, we expect $\delta_1 < 0$.

We construct four proxies for financial constraints, each based on firms' 2014 balance–sheet characteristics. Firms with a book value of total assets below the cross–sectional median, or with a book value of physical capital below the median, are classified as constrained. We also classify as constrained those with leverage above the median or with a cash–to–assets ratio below the median. All medians are computed using 2014 data to ensure that classification is predetermined with respect to treatment. Results are reported in Table IV.

The patterns are strikingly consistent across all measures. Firms that were not financially constrained prior to the introduction of the subsidy show no detectable change in their capital–to–intangible ratio. This aligns with the interpretation that these firms were already close to the frictionless input mix and thus faced negligible wedges. By contrast, treated firms that were financially constrained exhibit sizable reductions in the ratio relative to control firms. Treated firms with low asset levels reduce their capital–to–intangible ratio by 11.5 percent; those with high leverage reduce it by roughly 12 percent.

Taken together, these results reinforce the view that financial frictions distort firms' capital allocation and that the investment subsidy was particularly effective for the firms most affected by these distortions. The heterogeneity in treatment effects therefore provides an additional piece of evidence—complementary to the distributional analysis—that financial frictions, rather than relative–price movements, drive the observed rebalancing toward intangible capital.

Taken together, these results strengthen our interpretation of the aggregate evidence. The regional analysis rules out explanations based on relative price changes, while the firm-level results confirm that financial constraints drive the observed reallocation. The shift toward intangible capital therefore reflects a relaxation of financing wedges rather than differences in production technology or market prices.

3.4 Effects on the Levels of Physical and Intangible Capital

Thus far, we have focused on how firms adjust the composition of capital—specifically, the trade-off between physical and intangible assets. We now turn to the levels of these two inputs to assess whether the observed change in composition reflects substitution between capital types or an expansion in overall investment. To this end, we re-estimate equation (6), replacing the dependent variable with the logarithm of the stock of physical capital or the logarithm of the stock of intangible capital. The results are presented in Figure 5.

Before discussing the results, it is important to note that identification in this exercise is weaker than in the analysis of capital composition. Studying input levels, rather than their ratio, requires the standard assumptions of the misallocation literature: comparable production technologies and stable input price levels across firms.[11] In contrast, our previous analysis of the capital-to-intangible ratio relied only on relative price stability and did not require such assumptions. We therefore interpret the following results as suggestive evidence on the direction of adjustment, rather than as precise estimates of changes in physical or intangible quantities.

Following the introduction of the investment subsidy, treated firms show no significant change in the level of physical capital relative to control firms. By contrast, the stock of intangible capital rises markedly: between 2014 and 2018, treated firms increase their intangible capital by 11 percent—equivalent to roughly 51,000 euros—relative to control firms. This finding confirms that the decline in the capital-to-intangible ratio is driven by an expansion in intangible capital rather than by a contraction in physical capital.

The results in Figure 5 also display pre-trends in both types of capital, likely reflecting common shocks such as asset price inflation, productivity growth, or demand fluctuations. Analyzing the ratio of physical to intangible capital, as in our baseline specification, helps mitigate these confounding factors by differencing out common trends and isolating changes in relative input use. This distinction highlights why the ratio-based analysis provides a cleaner measure of the underlying wedge adjustments: it requires no assumptions about price levels, only about relative prices.

Finally, Online Appendix Figure C.4 explores regional heterogeneity in capital stock responses. We find no change in the stock of physical capital in either high- or low-subsidy regions. In contrast, treated firms in high-subsidy regions exhibit a clear increase in intangible capital relative to control firms, while treated firms in low-subsidy regions

[11] In particular, we must assume that price levels and output elasticities are similar across treated and control firms, so that changes in the measured capital stock reflect changes in quantities rather than valuation effects.

show no significant change. This pattern reinforces our earlier conclusion that substantial subsidies are necessary to induce adjustments in capital accumulation—and that these adjustments are concentrated in intangible assets.

3.5 The Extensive Margin of Adjustment

The analysis so far has combined both extensive and intensive margins of adjustment. By using the logarithm of the ratio of physical to intangible capital as the outcome variable, the estimated effects reflect two components: (i) the average change in capital mix among firms that adjust, and (ii) the share of firms that adjust. A large literature on capital adjustment frictions (Caballero et al., 1995; Khan and Thomas, 2008; Eberly et al., 2012; Ottonello and Winberry, 2023) emphasizes the importance of the extensive margin. We now isolate that margin in our setting.

Before proceeding, it is important to note that identification in this exercise is weaker than in our baseline specification. Studying the likelihood of adjustment requires stronger assumptions about price levels and production technologies across firms, similar to those in the misallocation literature. The results should therefore be interpreted as descriptive evidence on adjustment behavior rather than as precise measures of underlying wedges.

We estimate the following event–study specification:

$$\begin{aligned} \text{Adjusted}^h_{i,t} &= \alpha_i + \lambda_t + \beta X_{i,t} \\ &\quad + \sum_{m=-5,\, m\neq -1}^{4} \delta_m \, \mathbf{1}\{i \in \text{Treated}\} \, \mathbf{1}\{t = 2015 + m\} + \varepsilon_{i,t}, \end{aligned} \tag{8}$$

where the dependent variable $\text{Adjusted}^h_{i,t}$ equals one if firm i adjusts its stock of capital type $h \in \{\text{Physical, Intangible}\}$ in year t, and zero otherwise. We define adjustment as an absolute annual change in the capital stock exceeding one percent:

$$\text{Adjusted}^h_{i,t} = \mathbf{1}\left\{ \left| \frac{K^h_{i,t+1} - K^h_{i,t}}{K^h_{i,t}} \right| > 0.01 \right\}.$$

As before, we include firm and year fixed effects and a vector of time–varying controls. Standard errors are clustered at the firm level. The results are shown in Figure 6.

Following the introduction of the investment subsidy, treated firms are no more likely than control firms to adjust their stock of physical capital. In contrast, they are significantly more likely to adjust their stock of intangible capital. Between 2014 and 2019, the probability of adjusting intangible capital increases by 1.9 percentage points for treated firms relative to control firms—an increase of 2.6 percent relative to the baseline adjustment rate. This pattern is consistent with models of lumpy investment with fixed adjustment costs (Caballero et al., 1995; Khan and Thomas, 2008), in which only sufficiently large shocks trigger reoptimization.

In sum, the subsidy increases the incidence of adjustments in intangible capital while leaving physical capital adjustment rates unchanged. Together with our levels evidence (no change in physical capital, clear growth in intangibles), this indicates that the observed decline in the capital–to–intangible ratio reflects an expansion in intangible investment rather than a contraction of physical capital, without apportioning the total effect across margins.

3.6 Firm Performance

We have shown that the investment tax subsidy led firms to increase investment in intangible assets and shift their capital mix away from physical capital. Our working hypothesis is that, prior to the subsidy, a wedge between the marginal rate of technical substitution and the price ratio distorted capital allocation—specifically, that the marginal return to intangible capital was inefficiently high. If this wedge is reduced by the policy, and firms increase investment in high-return intangible assets, firm performance should improve.

To test this implication, we estimate the following difference-in-differences specification:

$$\log Y_{i,t} = \lambda_t + \alpha_i + \beta X_{i,t} + \delta \cdot \mathbf{1}\{i \in \text{Treated}\} \cdot \mathbf{1}\{t \geq 2015\} + \varepsilon_{i,t}, \tag{9}$$

where the outcome variable $Y_{i,t}$ is one of five variables: (1) cash holdings, (2) the ratio of cash holdings to assets, (3) debt, (4) short-term debt, and (5) long-term debt. The regression includes firm fixed effects, year fixed effects, and a vector of time-varying firm controls. Standard errors are clustered at the firm level. The coefficient of interest, δ, captures the average treatment effect of the investment tax subsidy. The results are reported in Table V.

Treated firms increase their cash holdings by 2 percent relative to control firms, corresponding to an average increase of nearly 6,000 euros. This suggests that firms do not immediately invest the full amount of the subsidy; instead, they partially retain it. This behavior is consistent with models of investment subject to fixed adjustment costs, where firms stockpile liquidity in anticipation of future investment episodes. It also aligns with the pecking order theory of Myers and Majluf (1984), in which firms prioritize internal financing. We also observe that the increase in cash holdings is larger than the increase in assets, as the ratio of cash holdings to assets increases by 2 percentage points for treated firms relative to control firms.

Firms also adjust their debt structure. Treated firms increase overall debt by 2.4 percent relative to firms in the control group. Treated firms reduce their short-term debt by 7 percent (approximately 45,000 euros), while increasing long-term debt by 6 percent (around 83,500 euros) relative to control firms. Although total debt increases, its maturity shifts toward the long term. This is consistent with firms becoming more forward-looking and facing looser financial constraints after the policy.

We also study the effect of the investment subsidy on firm performance. We estimate equation (9) for four outcome variables: 1) the logarithm of sales, (2) the logarithm of operating profits (sales minus operating costs), (3) the ratio of operating profits to sales, and (4) an indicator variable that takes the value of one if the firm makes strictly positive operating profits, and zero if otherwise. We present the results in Table VI.

Sales of treated firms increase by 3 percent as a result of the investment subsidy when compared to firms in the control group. Operating profits increase by 1.4 percent relative to control firms—equivalent to a gain of 22,475 euros compared to the unconditional mean. However, there is no evidence that the investment subsidy increased the profit margin of treated firms relative to control firms. The likelihood of generating strictly positive operating profits also rises: treated firms are 0.2 percentage points more likely to be profitable, a 0.2 percent increase relative to control firms.

Taken together, these results support the interpretation that the investment tax subsidy reduced capital misallocation and improved firm outcomes, both through investment and balance sheet channels.

4 Effects on the Distribution of the Capital-to-Intangible Ratio

The event–study analysis in Section 3 focused on average treatment effects, capturing how the typical firm adjusted its capital mix in response to the investment subsidy. While informative, those results mask the underlying heterogeneity in firms' responses — heterogeneity that is central to understanding how financial frictions shape input choices. This section moves beyond averages to study how the entire distribution of the capital-to-intangible ratio evolves after the policy. By tracing the full distribution rather than its mean, we can assess whether the subsidy primarily affected constrained firms, whether distortions compressed across the economy, and ultimately, how much misallocation the policy alleviated.

4.1 Empirical Approach: The Change-in-Changes Design

So far, we have shown that, on average, firms exposed to the investment subsidy shift their input mix toward intangible assets. We interpret this reallocation as the result of financial frictions being relaxed by the liquidity shock created by the subsidy. Yet average effects can conceal substantial heterogeneity. For instance, if the decline in the mean capital-to-intangible ratio were driven entirely by firms already at the lower end of the distribution, this would be inconsistent with our interpretation — since such firms are less likely to be financially constrained. Understanding how the entire distribution shifts is therefore essential.

To study distributional effects, we use the *Change-in-Changes* (CiC) framework of Athey and Imbens (2006) and Chernozhukov et al. (2013). The CiC estimator extends the logic of difference-in-differences from means to distributions. In standard difference-in-differences, identification relies on the assumption that, absent treatment, the *average* outcome for treated firms would have evolved in parallel with that of control firms. Under this assumption, we can construct a counterfactual path for the mean and obtain the average treatment effect. CiC makes an analogous assumption for the *entire distribution*: absent treatment, the *rank* of each treated firm within its outcome distribution would evolve through the same monotone transformation observed among control firms. This assumption — known as *monotone rank preservation* — allows us to recover counterfactual distributions and estimate *quantile treatment effects* (QTEs) conditional on a firm's initial rank in the outcome distribution.

Formally, for each percentile τ in the initial distribution of outcomes (the distribution of the logarithm of the capital-to-intangible ratio for treated firms in 2014), we compute the QTE as

$$\mathrm{QTE}_t(\tau) = Q_t^T(\tau) - \underbrace{h_t(Q_{2014}^T(\tau))}_{\text{counterfactual}}, \tag{10}$$

where $h_t(\cdot)$ is a monotone trend mapping defined implicitly by

$$F_t^C(y) = F_{2014}^C(h_t^{-1}(y)),$$

and Q denotes the inverse quantile function. Intuitively, $h_t(\cdot)$ describes how each point in the control group's outcome distribution moves over time. The first step is to estimate this monotone transformation from the control group. The second step assumes that, in the absence of treatment, treated firms would have followed the same transformation. The third step compares the observed outcome of treated firms at each quantile to this counterfactual to obtain $\text{QTE}_t(\tau)$.

The identifying assumptions behind CiC are stronger than those required for mean difference-in-differences. In addition to monotone rank preservation, CiC requires a stronger version of the common-trend assumption — namely, that the untreated outcome distributions for treated and control firms evolve through the same rank-preserving transformation. In our context, this means that, absent the subsidy, any evolution in the capital-to-intangible ratio would have shifted the entire distribution of treated firms in the same way as that of controls.

The intuition is straightforward. In difference-in-differences, we assume that treated and control firms would have experienced the same *shift* in their mean outcomes. In CiC, we instead assume that they would have experienced the same *reshaping* of their entire distributions. This assumption rules out systematic differences in the distribution of idiosyncratic shocks or cyclical exposures that could change firms' relative positions in the outcome distribution. For example, if treated firms drew idiosyncratic shocks from a different distribution, their relative ranks in the capital-to-intangible ratio could change even in the absence of treatment, violating rank preservation. In such cases, mean-independence would still hold and a difference-in-differences design would correctly estimate the average treatment effect, but ranks would not be stable, and CiC would incorrectly attribute these distributional shifts to the policy. However, in most models, relative prices are identical across firms — or at least drawn from the same distribution — making the CiC assumptions plausible in our setting.

For this reason, the quantile treatment effects we estimate should not be interpreted as structural parameters. However, the overall *shape* of the QTE distribution remains highly informative. If the policy primarily relaxes financial constraints, we should observe the largest effects among firms initially at the top of the capital-to-intangible distribution — those with the highest distortions and strongest reallocation potential. The precise magnitudes should be interpreted with caution, but the qualitative pattern of heterogeneity provides an important test of the underlying mechanism.

4.2 Distributional Effects: Change-in-Changes Results

To examine how the distribution of the logarithm of the capital-to-intangible ratio evolves after the policy, we apply the CiC estimator for all years $t \geq 2015$, using 2014 as the reference year. Inference is obtained through a nonparametric block bootstrap that resamples firms—rather than individual observations—with replacement.[12] We focus on 2017 as a representative post-treatment year, by which time the effects of the policy are fully reflected in firms' balance sheets but the program remains in force. The results of this estimation are shown in Figure 7.

The quantile treatment effects display a clear and economically meaningful pattern consistent with the presence of financial frictions. Firms in the lower percentiles of the log capital-to-intangible ratio distribution—those already close to the frictionless input mix—show no statistically significant change following the introduction of the subsidy. In contrast, firms in the upper part of the distribution, which are further from the unconstrained input mix and thus more likely to be financially constrained, exhibit a substantial decline in the ratio. For instance, a firm initially at the 75th percentile experiences a roughly 12 percent reduction in its capital-to-intangible ratio relative to a comparable control firm.[13] This heterogeneous response across the distribution reinforces our earlier conclusion: the policy primarily relaxed financing constraints rather than altering relative input prices.

The downward-sloping pattern of the quantile treatment effects implies a compression in the distribution of wedges. Under our identifying assumptions, changes in the capital-to-intangible ratio can be interpreted as changes in the wedge between a firm's observed and frictionless input mix. The investment subsidy thus reduced overall distortions by narrowing the spread of wedges across firms. Most of this compression arises from a reduction in wedges among firms in the upper tail of the distribution—those that were initially the most constrained. While these effects should be interpreted cautiously, they suggest that the policy improved allocative efficiency by bringing the most distorted firms closer to their unconstrained equilibrium.

To visualize this compression directly, we compute the gap between the post-treatment

[12] Because treatment is assigned at the firm level and outcomes are observed over time, resampling entire firm histories preserves the within-firm serial dependence structure of the data. This procedure is conceptually analogous to clustering standard errors at the firm level: by treating each firm as the resampling unit, we allow for arbitrary autocorrelation and heteroskedasticity within firms while maintaining independence across firms. This approach ensures inference that is robust to serial dependence, which is essential in event–study and CiC settings.

[13] Results for all post-treatment years are reported in Online Appendix Figure C.5. The same qualitative pattern—null effects for low percentiles and negative effects for high percentiles—persists throughout the sample and becomes more pronounced over time.

and counterfactual distributions implied by the CiC estimator. For each percentile τ, the CDF gap is defined as

$$G_t(y) = \mathbb{P}(Y_t^T \leq y) - \mathbb{P}(Y_t^{\text{CF}} \leq y),$$

which measures the difference between the empirical distribution of treated firms in year $t \geq 2015$ and the counterfactual distribution predicted in the absence of treatment. Figure 8 plots this gap for the post-treatment period.

The CDF gap provides a complementary visualization to the QTE plot. While the QTEs capture how treatment effects vary across percentiles, the CDF gap shows how probability mass shifts across the entire distribution. The gap is negative at low values and positive at high values, with a single crossing point—indicating that mass shifted from the upper tail toward the center. In economic terms, there are fewer firms with large wedges and more firms with moderate ones. This pattern confirms that the subsidy compressed the wedge distribution, primarily benefiting the most constrained firms. The single crossing further suggests a first-order stochastic dominance improvement, consistent with a more efficient allocation of inputs across firms.[14]

Having documented these distributional shifts, we next translate the estimated quantile-level effects into a distribution of wedges that rationalizes the data under our model.

4.3 Recovering the Distribution of Wedges

In Section 3, we used the average treatment effect to compute a lower bound on the average wedge in the economy prior to the introduction of the investment subsidy. We now go further and recover the entire *distribution* of wedges for the least distorted economy that is consistent with our empirical results.[15]

Intuitively, our goal is to characterize the minimum degree of misallocation compatible with the data. Because the wedge ω is bounded below by one, we can construct for each percentile τ a lower bound $\tilde{\omega}(\tau)$ on the wedge as

$$\tilde{\omega}(\tau) = \max\left\{1, \exp\left\{-\min_{t\geq 2015} \text{QTE}_t(\tau)\right\}\right\}. \tag{11}$$

For any percentile τ in which the quantile treatment effect is negative for all post-treatment

[14]This distributional compression mirrors the aggregate decline in the average wedge documented earlier, reinforcing the interpretation that the policy alleviated financing distortions rather than altering relative prices.

[15]We define the *least distorted economy* as the economy in which, for each firm, the wedge equals the lower bound implied by the quantile treatment effects reported in Figure 7.

years, the minimum of $\text{QTE}_t(\tau)$ corresponds to the largest decline in the wedge experienced by firms initially in that percentile. Because $\omega \geq 1$, this minimum identifies a sharper lower bound than the trivial value of one.[16] Conversely, for percentiles where the quantile treatment effect is non-negative, the lower bound remains one, as the data do not provide additional information about reductions in distortions.

Equation (11) provides a lower bound for each percentile τ. To transform this mapping into a full distribution of wedges, we exploit the fact that the percentile rank τ of the underlying variable (the logarithm of the capital-to-intangible ratio) is uniformly distributed on the unit interval.[17] We observe $\tilde{\omega}(\tau)$ on a fine grid of percentiles and linearly interpolate between grid points to approximate the continuous function. Recovering the distribution $F^{\star}(\tilde{\omega})$ of wedges from the estimated quantile curve is a standard change-of-variables problem—closely related to constructing distributions from quantile functions (Serfling, 2009). Full details of the algorithm are provided in Appendix D.

We use this algorithm to compute the distribution $F^{\star}(\tilde{\omega})$ of the minimum wedges that rationalize the data under our theoretical structure. By construction, this distribution represents the least distorted economy pointwise—firm by firm—because it takes the minimum wedge consistent with the evidence. The resulting distribution is shown in Figure 9.

The recovered distribution features a discrete mass at $\omega = 1$, the lower bound of the support. Approximately five percent of firms are located at this point, implying that they are effectively unconstrained. Hence, at most five percent of firms in our sample operate without financial frictions, suggesting that the Portuguese corporate sector is substantially distorted in terms of the marginal rate of technical substitution. The median wedge is 1.13, almost identical to the lower bound for the average wedge reported in Section 3. Moreover, distortions are quantitatively significant: twenty-five percent of firms exhibit wedges of at least 1.2, meaning their capital-to-intangible ratios are at least twenty percent higher than the frictionless benchmark.

The distribution $F^{\star}(\tilde{\omega})$ therefore provides the tightest possible lower bound on misallocation consistent with the data. Any other distribution supported on the same domain that first-order stochastically dominates $F^{\star}(\tilde{\omega})$ is also consistent with the data and the model.[18] In this sense, the economy characterized by $F^{\star}(\tilde{\omega})$ is the *least distorted* allocation compatible with our results. Any empirically or theoretically plausible alternative

[16] Since the outcome variable is expressed in logs, the transformation from log changes to level changes is exponential.

[17] This follows directly from the probability integral transform (Feller, 1991).

[18] To see this, suppose that wedges are instead given by $\omega' = \omega + \eta$, where $\eta \geq 0$ a.s.. The new set of wedges is weakly larger almost surely, which represents a more constrained economy. Moreover, it follows

economy must be weakly more distorted—that is, must first-order stochastically dominate $F^{\star}(\tilde{\omega})$. This construction transforms the local evidence from the investment subsidy into a global statement about the minimum degree of misallocation in the economy.

This exercise also connects our analysis to two broader literatures. First, it provides a micro–empirical bridge to the misallocation literature (Hsieh and Klenow, 2009), which typically infers wedges indirectly from output and input shares under strong assumptions about technology and prices. In contrast, our approach recovers the entire distribution of wedges directly from observable firm behavior, requiring minimal structural assumptions. Second, it relates naturally to the wedge–accounting tradition in macroeconomics (Chari, Kehoe and McGrattan, 2007), which interprets aggregate inefficiencies as arising from implicit distortions in firms' first–order conditions. Our results can thus be viewed as a micro–level analogue of this framework: we recover the empirical distribution of wedges that rationalizes firm–level choices, quantify its dispersion, and identify the least distorted allocation consistent with the data. By doing so, we provide a direct measure of the magnitude and distributional shape of financial distortions in the economy.

Finally, these findings complete the narrative begun in the event–study analysis. The average decline in the capital-to-intangible ratio reflected an economy-wide easing of financial constraints; the distributional evidence now reveals that this adjustment was concentrated among the most distorted firms. By recovering the least distorted distribution of wedges, we quantify not only how much the policy improved efficiency on average, but also how it reshaped the dispersion of distortions across firms.

5 Conclusion

This paper shows that financial frictions distort not only the allocation of resources across firms but also the composition of investment within firms. Exploiting a quasi-experimental policy that increased firms' liquidity without altering relative prices, we find that treated firms systematically rebalanced their input mix toward intangible capital. This response is inconsistent with a frictionless model but follows naturally from a framework in which financing constraints raise the effective cost of intangible investment. By linking a clean empirical design to a transparent theoretical structure, we provide direct evidence that

that

$$F_{\omega'}(x) = \mathbb{P}\left(\omega + \eta \leq x\right) = \mathbb{P}\left(\omega \leq x - \eta\right) \leq \mathbb{P}\left(\omega \leq x\right) = F^{\star}(x),$$

and so $F_{\omega'}$ first order stochastically dominates $F^{\star}$.

financial frictions shape firms' capital composition.

Quantitatively, the investment subsidy reduced the wedge between the marginal rate of technical substitution and the relative price of inputs by about eleven percent, relaxing financial constraints and improving allocative efficiency. Heterogeneity across regions and firms reinforces this interpretation. The effects are strongest where the subsidy was more generous and among smaller firms, which are more likely to be constrained ex ante. The policy also translated into broader balance-sheet and real effects: treated firms accumulated more liquidity, shifted toward longer-maturity debt, and experienced higher operating profits.

Going beyond average effects, we show that the subsidy reshaped the entire distribution of distortions. The policy compressed the distribution of wedges, with the largest adjustments occurring among firms that were initially the most constrained. Using distributional methods, we recover quantile treatment effects across the full distribution of the capital-to-intangible ratio, revealing a monotone pattern consistent with the relaxation of financial frictions. Building on these estimates, we develop a new approach to recover the cross-sectional distribution of wedges for the least distorted economy consistent with the data.

The recovered distribution provides a tight lower bound on misallocation. At most five percent of firms appear unconstrained, while one quarter face wedges exceeding twenty percent. More generally, any economy consistent with the data must be weakly more distorted than this benchmark. These results connect firm-level responses to the broader misallocation and wedge-accounting literatures, offering a new way to measure both the magnitude and the distribution of financial distortions. By recovering the least distorted allocation compatible with observed behavior, our framework provides a disciplined empirical bridge between reduced-form evidence and structural accounts of aggregate productivity losses driven by financial frictions.

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Figures and Tables

FIGURE 1: Distribution of Capital-to-Intangible Ratio

This figure plots the kernel density of the logarithm of the capital-to-intangible ratio for all firms in our sample in 2014. We present the density for control firms and treated firms.

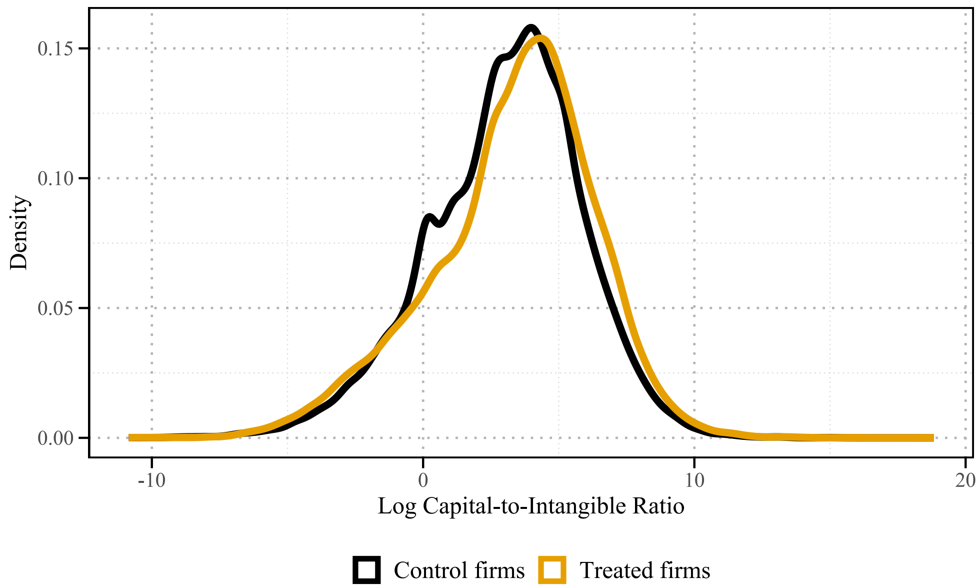


FIGURE 2: Effect on Aggregate Capital Stocks

This figure compares the evolution of the sum of the stock of physical capital and the sum of the stock of intangible capital for treated and control firms. We first aggregate these variables and then take the logarithms and scale by their values in 2014.

(a) Physical Capital

(b) Intangible Capital

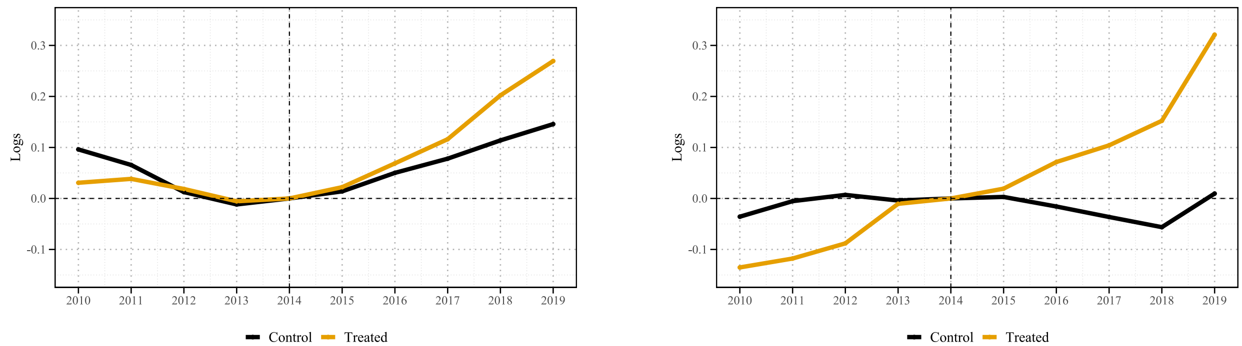


FIGURE 3: Effect on Firm-Level Capital-to-Intangible Ratio

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

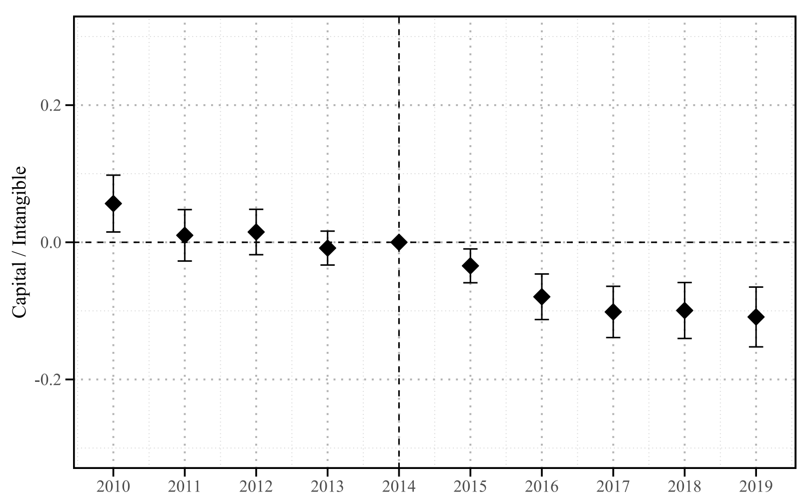


FIGURE 4: Effect on Firm-Level Capital-to-Intangible Ratio - Decomposition by Region

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We present results for three samples: (1) including all firms, (2) including only firms in regions where the investment subsidy was 10 percent, and (3) including only firms in regions where the investment subsidy was 25 percent. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

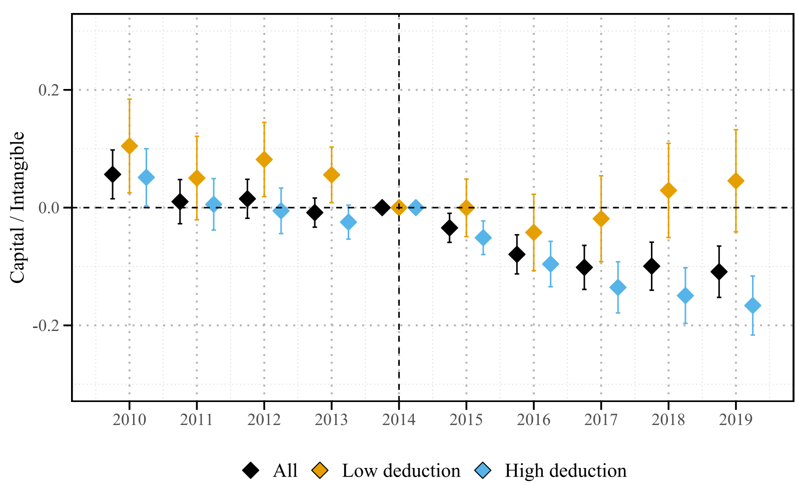
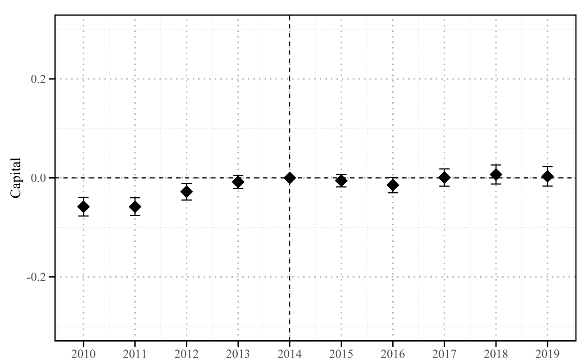


FIGURE 5: Effect on Firm-Level Capital Stocks

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is either the logarithm of the stock of physical capital at the end of the period or the logarithm of the stock of intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

(a) Physical Capital



(b) Intangible Capital

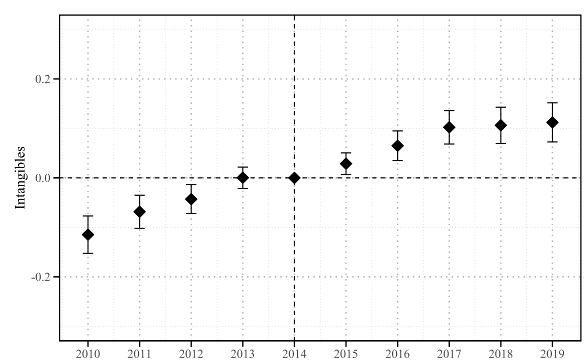
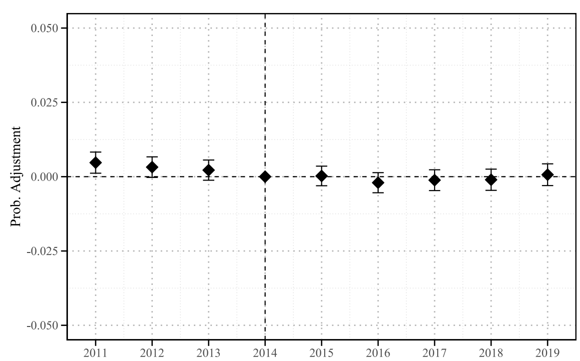


FIGURE 6: Effect on Firm-Level Probability of Adjustment

This Figure presents the results of estimating equation (8) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is either the an indicator that takes the value of 1 if the firm has adjusted its stock of physical capital and zero otherwise or an outcome variable that takes the value of 1 if the firm has adjusted its stock of intangible capital and zero otherwise. We define adjustment as having a change in the stock which is larger in absolute value than 1 percent of the lagged stock. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

(a) Physical Capital



(b) Intangible Capital

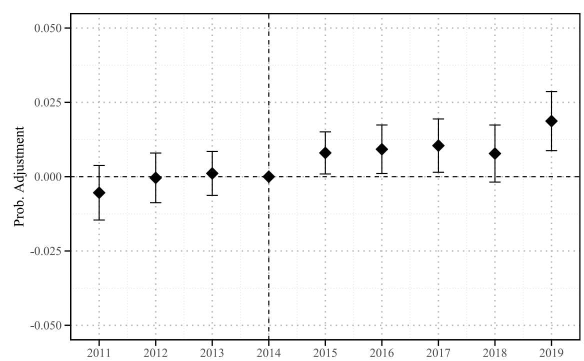


FIGURE 7: Effect on Distribution of Capital-to-Intangible Ratio

This figure presents the results of a Change-in-Changes estimation using a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. For each percentile τ of the distribution of the outcome variable for treated firms in 2014 (the base year), we estimate the quantile treatment effect (QTE) between 2017 and the base year of 2014. We compute standard errors via a nonparametric block bootstrap that resamples firms (rather than individual observations) and present 95 percent confidence intervals.

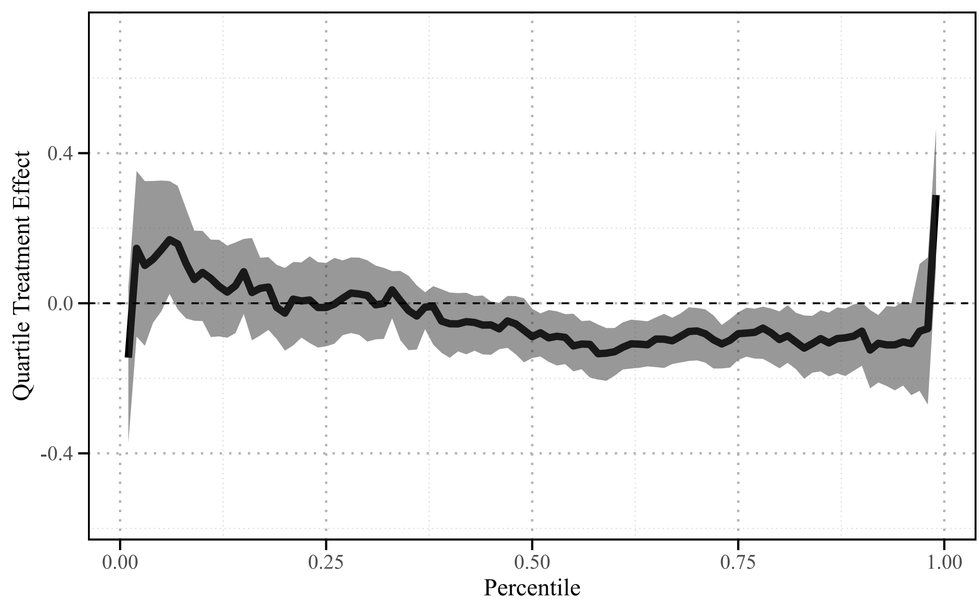


FIGURE 8: Distribution Gap

This figure presents the results of a Change-in-Changes estimation using a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We present the difference between the distribution of outcomes for treated outcomes in 2017 and the counterfactual distribution for the same year computed using the CiC estimator.

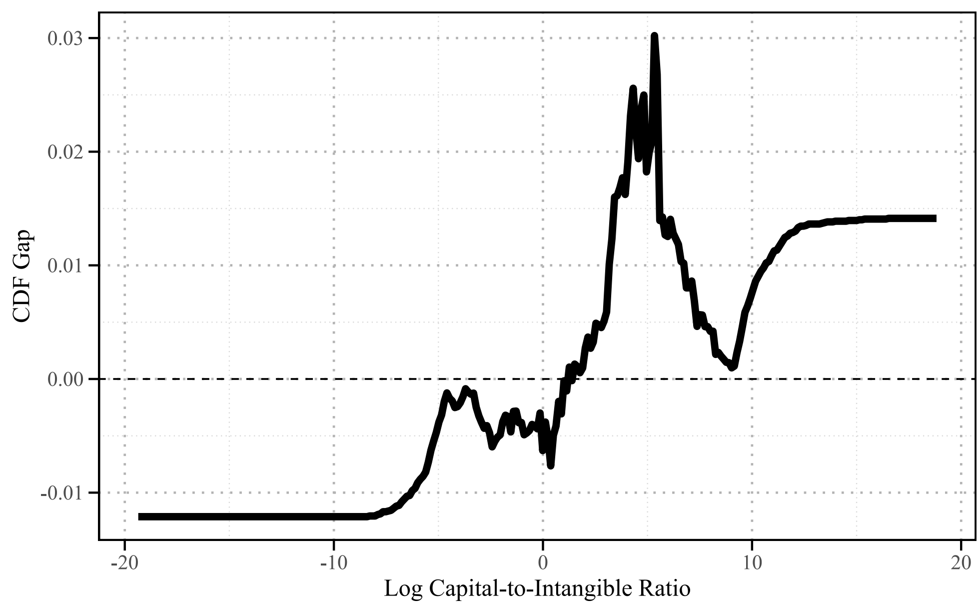


FIGURE 9: Distribution of Wedges in Least Distorted Economy

This figure presents the estimated distribution of wedges in the least distorted economy which is consistent with the empirical evidence.

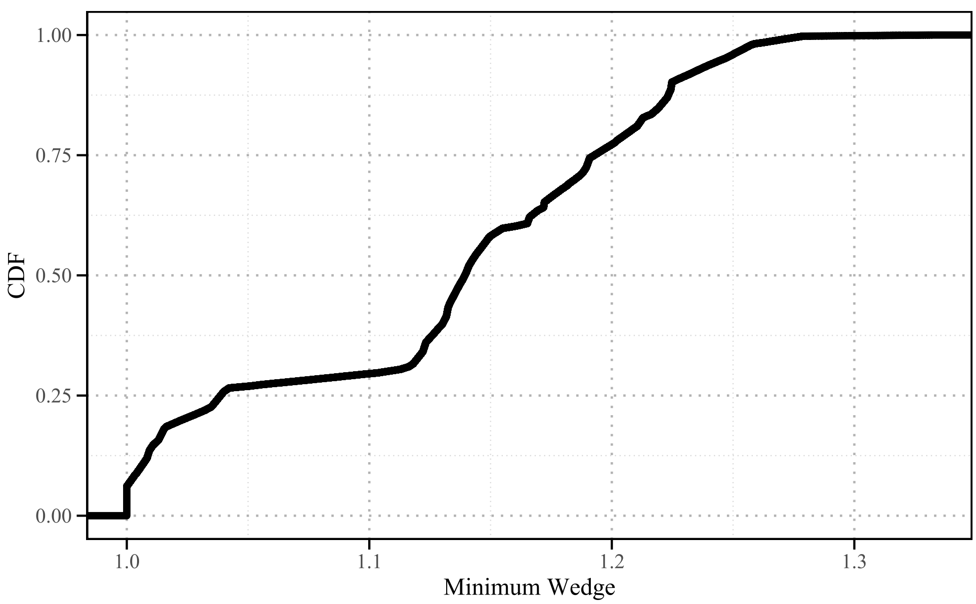


TABLE I: Cash Generated by Operations

This table presents the impact of the investment subsidy on a firm. The firm carries out an investment of 20 in either physical or intangible capital in year 0. The asset fully depreciates in five years, and so the added depreciation is 4. The tax rate is 30 percent and the investment subsidy is 25 percent.

Year:	0	1	2	3	4	5
Income statement						
Sales	60	60	60	60	60	60
Operating costs	30	30	30	30	30	30
Investment costs	20	0	0	0	0	0
Depreciation	5	9	9	9	9	9
Cash flow						
Investment subsidy	5	0	0	0	0	0
Tax benefit from depreciation	1.5	2.7	2.7	2.7	2.7	2.7
Free cash flow from operations	7.5	23.7	23.7	23.7	23.7	23.7

TABLE II: Summary Statistics

This table presents summary statistics for all firms in our sample in 2014. We consider two groups: all firms, firms in the treated group, and firms in the control group. For each group we present the cross-sectional average of: total assets, total sales, the number of workers, the wage bill, the leverage ratio (measured as the ratio of total liabilities to total assets), the ratio of cash to total assets, the ratio of EBITDA to total sales, tangible assets (measured using PP&E), and intangible assets. We also present the difference in means between treated firms and control firms. For each difference, we compute the t-statistic and present the significance level of the difference where ***, **, and * denote significance at the 0.1, 1, or 5 percent level, respectively.

	All firms	Treated	Control	Difference
Assets	1,597,737	1,450,813	1,644,305	-193,392
Sales	987,161	1,248,426	904,350	344,076*
Number of workers	8.13	11.92	6.93	5.00***
Wage bill	139,068	200,572	119,573	80,998***
Leverage ratio	3.30	4.02	3.07	0.94
Cash / Total assets	0.14	0.16	0.13	0.03
EBITDA / Sales	-0.63	-0.68	-0.62	-0.06
Tangible assets	318,396	469,059	270,641	198,414***
Intangible assets	98,442	30,596	119,946	-89,350***
Number of firms	311,731	75,026	236,705	-

TABLE III: Variance Decomposition of Capital-to-Intangible Ratio

This table presents the results of a variance decomposition where the outcome variable is the logarithm of the capital-to-intangible ratio. We use data up to 2014. We present the R^2 of the regression, which represents the share of the variation that is explained by the fixed effects.

	(1)	(2)	(3)	(4)
R^2 (%)	15.86	16.39	92.80	92.80
Sector FE	✓			
Sector-Year FE		✓		
Firm FE			✓	✓
Year FE				✓
Observations	263,018	263,018	263,018	263,018

TABLE IV: Effect on Firm-Level Capital-to-Intangible Ratio - Role of Financial Frictions

This Table presents the results of estimating equation (7). The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We consider four definitions of financially constrained firms: (1) firms with a book value of assets below the median, (2) firms with a book value of physical capital below the median, (3) firms with a leverage ratio above the median, and (4) firms with a cash-to-assets ratio below the median. The cross-sectional median is computed using observations in 2014 and firms are classified based on the data in 2014. We cluster errors at the firm level. ***, **, and * denote significance at the 0.1, 1, or 5 percent level, respectively.

	Assets	PPE	Leverage	Cash/Assets
Post x Treated	-0.038	-0.039*	-0.020	-0.030
	(0.023)	(0.022)	(0.023)	(0.025)
Post x Treated x Financially Constrained	-0.076**	-0.093***	-0.100***	-0.057*
	(0.033)	(0.034)	(0.033)	(0.034)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Firm controls	✓	✓	✓	✓
Observations	306,005	306,005	306,005	306,005
R^2	0.87	0.87	0.87	0.87

TABLE V: Effect on Firm-Level Debt

This Table presents the results of estimating equation (9). The outcome variable is one of the following five variables: (1) the logarithm of cash holdings, (2) the ratio of cash holding to total assets, (3) the logarithm of total debt, (4) the logarithm of short-term debt, and (5) the logarithm of long-term debt. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets (except for the first two outcome variables), the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effect. We cluster errors at the firm level. ***, **, and * denote significance at the 0.1, 1, or 5 percent level, respectively.

	Cash	Cash/Assets	Debt	ST Debt	LT Debt
Post x Treated	0.036***	0.024***	0.024**	-0.068***	0.058***
	(0.012)	(0.008)	(0.012)	(0.020)	(0.013)
Firm FE	✓	✓	✓	✓	✓
Time FE	✓	✓	✓	✓	✓
Firm controls	✓	✓	✓	✓	✓
Observations	445,275	451,971	339,471	188,736	288,316
R^2	0.84	1.00	0.91	0.88	0.91

TABLE VI: Effect on Firm-Level Performance

This Table presents the results of estimating equation (9). The outcome variable is one of the following four variables: (1) the logarithm of sales, (2) the logarithm of operating profits (sales minus operating costs), (3) the ratio of operating profits to sales, and (4) an indicator variable that takes the value of one if the firm makes strictly positive operating profits, and zero if otherwise. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effect. We cluster errors at the firm level. ***, **, and * denote significance at the 0.1, 1, or 5 percent level, respectively.

	Sales	Operating Profits	Operating Margin	Positive Profits
Post x Treated	0.030***	0.041***	0.022	0.0019*
	(0.006)	(0.006)	(0.017)	(0.0008)
Firm FE	✓	✓	✓	✓
Time FE	✓	✓	✓	✓
Firm controls	✓	✓ ✓	✓	
Observations	451,971	446,594	451,971	470,423
R^2	0.95	0.94	0.49	0.44

Online Appendix

A Summary Statistics

FIGURE A.1: Share of Treated Firms

This Figure presents the the share of treated firms in 2014 among all firms. We consider six quantities: number of workers, total sales, total wage bill, stock of physical capital (measured using PP&E), total assets, and the number of firms.

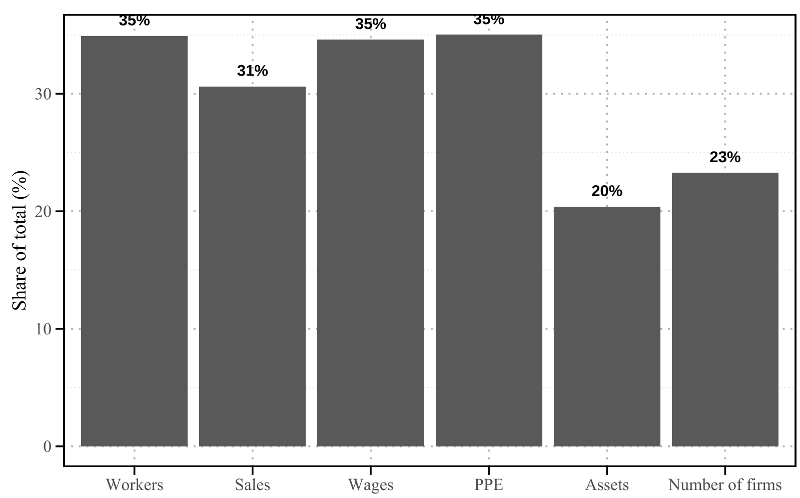


FIGURE A.2: Share of Firms in Regions with a High Deduction

This Figure presents the the share of firms in regions with a high deduction among all treated firms in 2014. We consider six quantities: number of workers, total sales, total wage bill, stock of physical capital (measured using PP&E), total assets, and the number of firms.

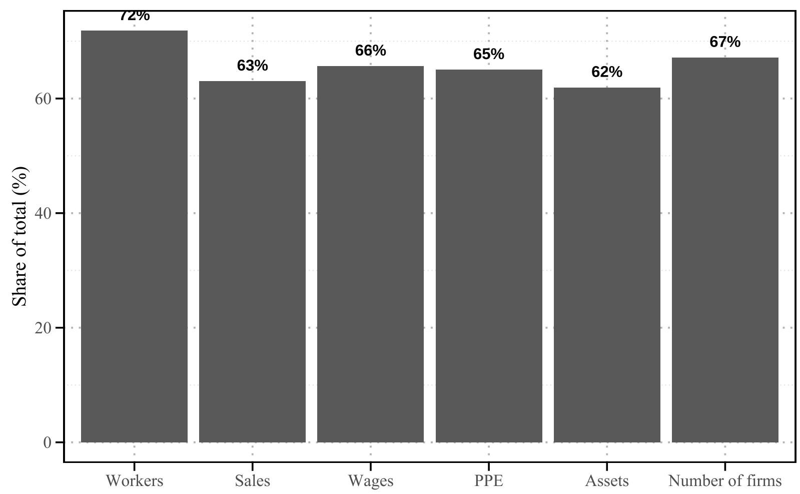


FIGURE A.3: Share of Firms in Regions with a High Deduction

This figure compares the evolution of the aggregate ratio of physical capital to intangible capital for treated and control firms. We first aggregate these variables, compute the ratio, and then take the logarithms and scale by their values in 2014.

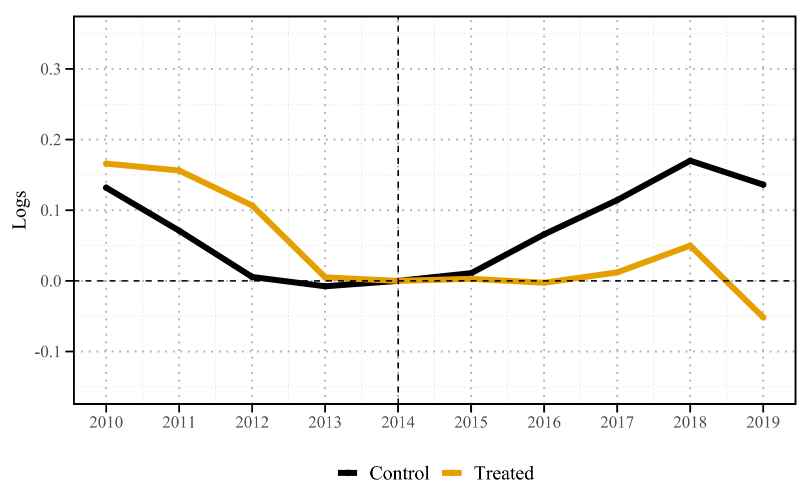
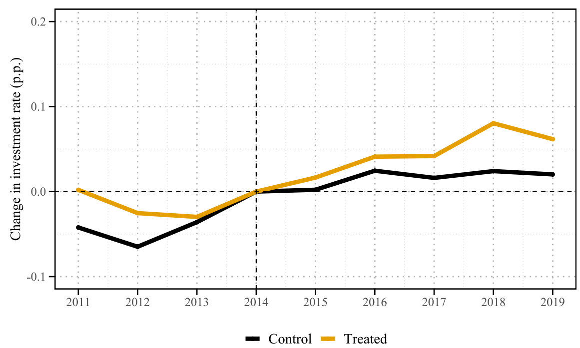


FIGURE A.4: Effect on Aggregate Investment

This figure compares the evolution of the aggregate investment in physical capital and intangible capital for treated and control firms. We first aggregate the stock of both type of capital for both types of firm and then compute the change in logs. We then subtract the level of investment in 2014.

(a) Physical Capital



(b) Intangible Capital

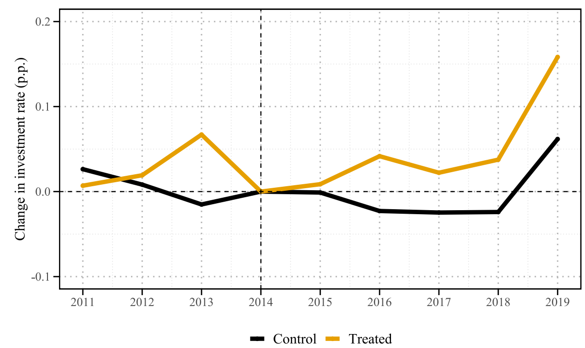
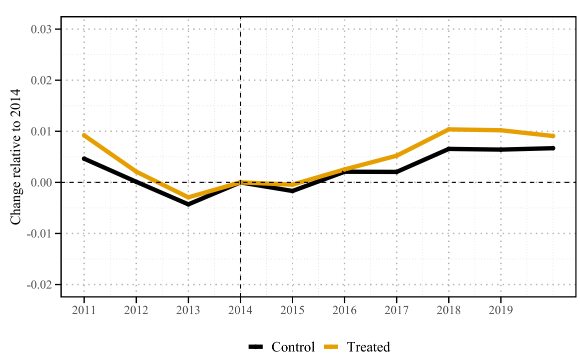


FIGURE A.5: Effect on Aggregate Probability of Adjustment

This figure compares the evolution of the aggregate probability of investing in either physical capital or intangible capital for treated and control firms. For each group of firms, we compute the share of firms that invest more than 1 percent in absolute value, relative to the lagged stock. We then subtract the level of this variable in 2014 to scale the time series.

(a) Physical Capital



(b) Intangible Capital

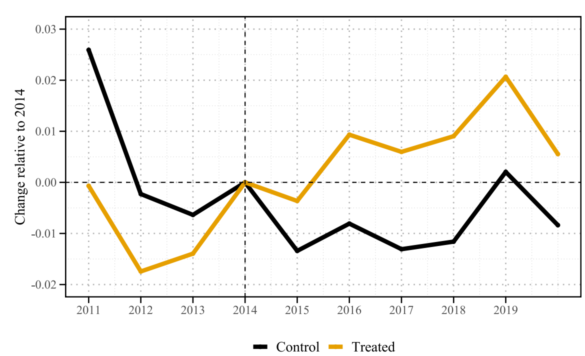


FIGURE A.6: Effect on Different Types of Intangibles

This Figure compares the evolution of different types of intangible assets for treated and control firms. We consider four different types of intangible capital - goodwill, R&D, patents, and other intangibles. For each type of intangible capital, we compute the total stock for treated and control firms. We then take the logarithm and subtract the value on 2014. We present the different between the time series for treated fimrs and the time series for control firms.

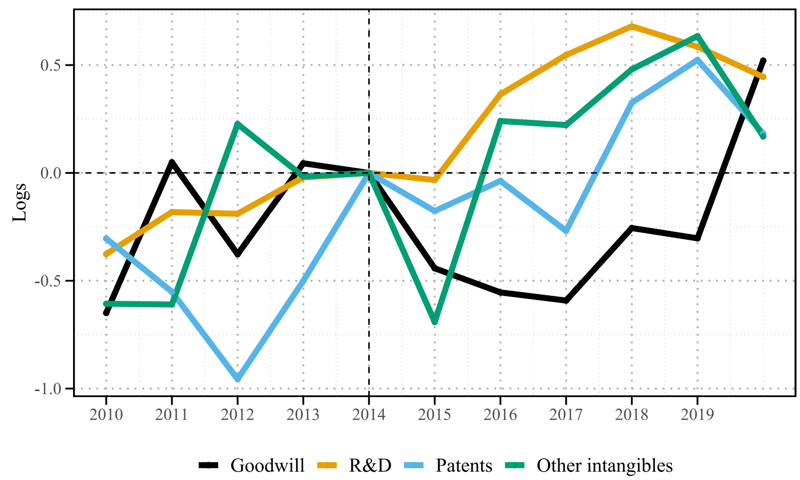


TABLE A.I: Treated Sectors

This table presents the list of treated sectors. For each sector, we also present the share of the number of firms, total assets, sales, or number of workers of that sector relative to all treated firms. All shares are in percentage. The treated sectors are: (1) mining (sector 0500 to 0999), (2) manufacturing (sector 1000 to 3399), (3) hotels & restaurants (sector 5500 to 5699), (4) editing services (sector 5800 to 5899 and sector 5910 to 5919), (5) IT services (sector 6200 to 6299 and sector 6310 to 6319), (6) R&D services (sector 7200 to 7299), and (7) other services (sectors 7721, 9004, 9104, 9311, 9321, 9329, 9329, 9604, 8211, and 8291). All sectors codes use the CAE Rev. 3 classification.

	Number of firms	Assets	Sales	Workers
Mining	0.92	2.11	1.00	0.97
Manufacturing	45.19	71.67	84.37	67.60
Hotels & Restaurants	40.25	16.61	8.27	22.90
Editing & Cinema	3.04	2.41	1.71	1.61
IT services	6.68	3.51	3.71	5.11
R&D	0.58	0.59	0.18	0.46
Other services	3.34	3.11	0.77	1.34

TABLE A.II: Summary Statistics - Treated Firms

This table presents summary statistics for all treated firms in our sample in 2014. We consider two groups: all treated firms, treated firms in regions with a high investment subsidy, and treated firms in regions with a low investment subsidy. For each group we present the cross-sectional average of: total assets, total sales, the number of workers, the wage bill, the leverage ratio (measured as the ratio of total liabilities to total assets), the ratio of cash to total assets, the ratio of EBITDA to total sales, tangible assets (measured using PP&E), and intangible assets. We also present the difference in means between treated firms in regions with a high tax rebate and treated firms in regions with a low tax rebate. For each difference, we compute the t-statistic and present the significance level of the difference where ***, **, and * denote significance at the 0.1, 1, or 5 percent level, respectively.

	All treated firms	High deduction	Low deduction	Difference
Assets	1,450,813	1,351,074	1,659,463	-308,390
Sales	1,248,426	1,168,227	1,416,199	-247,972
Number of workers	11.92	12.68	10.34	2.33***
Wage bill	200,572	194,668	212,921	-18,253
Leverage ratio	4.02	3.06	6.02	-2.97
Cash / Total assets	0.16	0.15	0.19	-0.04***
EBITDA / Sales	-0.68	-0.66	-0.72	0.06
Tangible assets	469,059	451,727	505,319	-53,592
Intangible assets	30,596	20,591	51,527	-30,836*
Number of firms	75,026	24,265	50,761	-

B Proofs

B.1 Proof of Proposition 1

B.1.1 Solution

Define $\rho = \frac{\sigma-1}{\sigma}$. Define also $Z = \alpha(\phi_k k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\phi_h h)^{\frac{\sigma-1}{\sigma}}$.

The firm solves

$$V(a) := \max_{k,h} A\left[\alpha(\phi_k k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\phi_h h)^{\frac{\sigma-1}{\sigma}}\right]^{\eta\frac{\sigma}{\sigma-1}} - p_k k - p_h h - \mu(p_h h - \lambda a)$$

and the first order conditions are given by

$$\begin{aligned} p_k &= \eta \cdot A \cdot \alpha \phi_k^{\rho} k^{\rho-1} \cdot Z^{\eta\frac{\sigma}{\sigma-1}-1} \\ p_h(1+\mu) &= \eta \cdot A \cdot (1-\alpha)\phi_h^{\rho} h^{\rho-1} \cdot Z^{\eta\frac{\sigma}{\sigma-1}-1} \\ \mu(p_h\lambda - \lambda a) &= 0. \end{aligned}$$

Using the first order conditions, we have

$$\begin{aligned} &\Leftrightarrow \frac{p_k}{p_h(1+\mu)} = \frac{\alpha}{1-\alpha}\left(\frac{\phi_k}{\phi_h}\right)^{\rho}\left(\frac{k}{h}\right)^{\rho-1} \\ &\Leftrightarrow \frac{p_k}{p_h(1+\mu)} = \frac{\alpha}{1-\alpha}\left(\frac{\phi_k}{\phi_h}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{k}{h}\right)^{-\frac{1}{\sigma}} \\ &\Leftrightarrow \left(\frac{k}{h}\right)^{\frac{1}{\sigma}} = \frac{\alpha}{1-\alpha}\left(\frac{\phi_k}{\phi_h}\right)^{\frac{\sigma-1}{\sigma}}\frac{p_h}{p_k}(1+\mu) \\ &\Leftrightarrow \frac{k}{h} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left(\frac{\phi_k}{\phi_h}\right)^{\sigma-1}\left(\frac{p_h}{p_k}\right)^{\sigma}(1+\mu)^{\sigma}. \end{aligned}$$

Finally, we can multiply both sides by relative prices to obtain

$$\frac{p_k k}{p_h h} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left(\frac{\phi_k}{\phi_h}\right)^{\sigma-1}\left(\frac{p_h}{p_k}\right)^{\sigma-1}(1+\mu)^{\sigma}.$$

B.1.2 Unique threshold

We want to prove two claims. First, that there exists unique threshold $\tilde{a}(A, p_k, p_h, \phi_k, \phi_h)$ such that for $a \geq \tilde{a}$, we have $\mu = 0$ and that for $a < \tilde{a}$, we have $\mu > 0$. Second, we want to show that, for $a \in (0, \tilde{a})$, $\mu(a)$ is a weakly decreasing function of a.

Notation. For each $a > 0$, let $\mathcal{K}(a) := \{(k,h) \in \mathbb{R}^2 : p_h h \leq \lambda a\}$ be the feasible set. Note that this set is nonempty, convex, and closed. Moreover, for $a_1 \leq a_2$, then $\mathcal{K}(a_1) \subseteq \mathcal{K}(a_2)$.

Let (k^u, h^u) be the maximizer of the unconstrained problem. Given the structure of the problem, this solution exists and is unique. Moreover, define $\tilde{a} = (p_h h^u)/\lambda$. Define also $(k^\star(a), h^\star(a))$ as the maximizer of the constrained problem, which is also unique.

Finally, define

$$\pi(k, j; a) \equiv A \left[\alpha(\phi_k k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(\phi_h h)^{\frac{\sigma-1}{\sigma}}\right]^{\eta \frac{\sigma}{\sigma-1}} - p_k k - p_h h.$$

Claim 1. If $a \geq \tilde{a}$, then (k^u, h^u) is feasible and is the unique maximizer. Therefore, the collateral constraint is slack and the firm is unconstrained.

Note that if $a \geq \tilde{a}$ then

$$a \geq \tilde{a} = \frac{p_h h^u}{\lambda} \implies \lambda a \geq p_h h^u \implies p_h h^u \leq \lambda a$$

and so $(k^u, h^u) \in \mathcal{K}(a)$ and is feasible.

Now, we need to show that (k^u, h^u) is the optimum. Suppose it is not. Instead, suppose that the optimum is $(k^\star(a), h^\star(a)) \neq (k^u, h^u)$ is the solution. However $(k^\star(a), h^\star(a))$ is also feasible as a solution for the unconstrained problem as $\mathcal{K}(a) \subseteq \mathcal{K}(\infty)$. But (k^u, h^u) is the unique maximizer of the unconstrained problem, which is a contradiction. Therefore, (k^u, h^u) is the optimum.

Finally, note that if $a > \tilde{a}$, the constraint is slack and $\mu = 0$. At $a = \tilde{a}$, the constraint holds with equality at (k^u, h^u) and the multiplier may also be zero.

Thus, for $a \geq \tilde{a}$, the unique solution is (k^u, h^u) and the firm is unconstrained.

Claim 2. If $a < \tilde{a}$, then the collateral constraint is binding.

In this region, $h^\star(a) = \lambda a / p_h$. First, note that by the definition of $\tilde{a}$

$$a < \frac{p_h h^u}{\lambda} \implies p_h h^u > \lambda a$$

and so the unconstrained choice is not feasible.

Now, we want to show that the collateral constraint is binding. Seeking a contradiction, suppose it does not. Let $(k^\star(a), h^\star(a))$ be the unique maximizer and assume that $p_h h^\star(a) < \lambda a$. Under this assumption, the Kuhn-Tucker conditions require that $\mu^\star(a) = 0$. However, in this case we recover the unconstrained problem, for which the unique solution is (k^u, h^u). However, this solution is not feasible, which is a contradiction. Therefore, the collateral constraint is binding.

Summary. We have shown that there is a unique threshold $\tilde{a}$ such that for $a \geq \tilde{a}$, we have $\mu = 0$ and that for $a < \tilde{a}$, we have $\mu(a) > 0$.

B.1.3 Multiplier is a decreasing function

We now want to show that $\mu(a)$ is a weakly decreasing function. Define the value function

$$V(a) = \max_{k,h \geq 0: p_h h \leq \lambda a} \pi(k,h).$$

We will first show that $V(a)$ is monotonically increasing and concave. First, note that if $a_1 \leq a_2$ then $\mathcal{K}(a_1) \subseteq \mathcal{K}(a_2)$ and therefore $V(a_2) \geq V(a_1)$.

We now want to show concavity. Let $a_1, a_2 \geq 0$ and let $\theta \in [0,1]$. Define $a_\theta = \theta a_1 + (1-\theta)a_2$. Let (k_1, h_1) be the solution to $V(a_1)$ and (k_2, h_2) be the solution to $V(a_2)$. This implies that $p_h h_1 \leq \lambda a_1$ and $p_h h_2 \leq \lambda a_2$. Define the convex combination $(k_\theta, h_\theta) = \theta(k_1, h_1) + (1-\theta)(k_2, h_2)$. Note that

$$p_h h_\theta = \theta p_h h_1 + (1-\theta) p_h h_2 \leq \theta \lambda a_1 + (1-\theta)\lambda a_2$$

and so this feasible. Therefore, it follows that $V(a_\theta) \geq \pi(k_\theta, h_\theta)$. Because the profit function is strictly concave (as the production function is strictly concave), we have that

$$\pi(k_\theta, h_\theta) \geq \theta\pi(k_1, h_1) + (1-\theta)\pi(k_2, h_2) = \theta V(a_1) + (1-\theta)V(a_2).$$

Therefore, it follows that

$$V(a_\theta) \geq \theta V(a_1) + (1-\theta)V(a_2)$$

and so V is concave.

Whenever V is differentiable (whenever $a \neq \tilde{a}$), the envelope theorem implies that

$$V'(a) = \mu(a)\lambda.$$

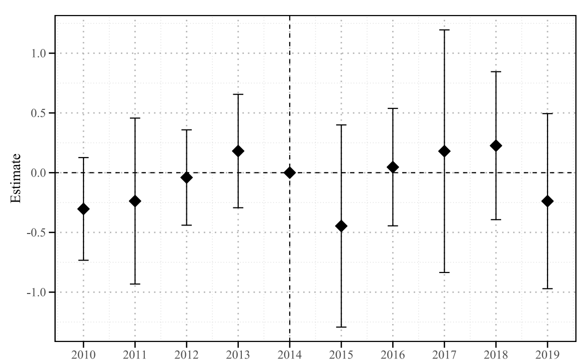
As V is concave, $V'(a)$ is weakly decreasing. Therefore, $\mu(a)$ is also weakly decreasing.

C Additional Firm-Level Results

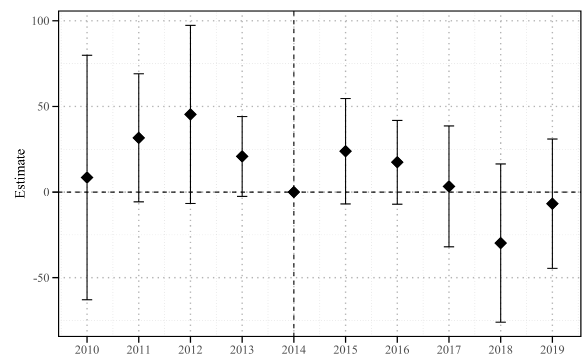
FIGURE C.1: Effect on Firm-Level SG&A

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. We consider three outcome variables: (1) sales and general administrative expenses (SG&A) scaled by assets, (2) SG&A scaled by the value of physical capital, and (3) SG&A scaled by the value of intangible capital. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

(a) SG&A/Sales



(b) SG&A/Physical Capital



(c) SG&A/Intangible Capital

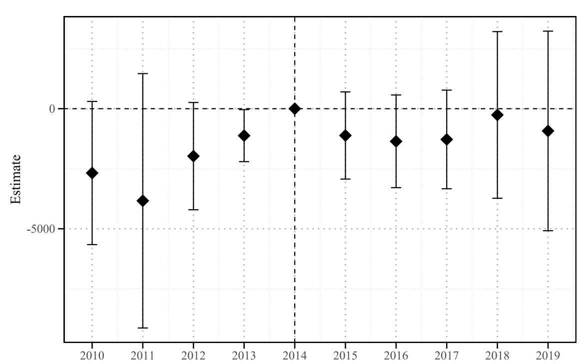


FIGURE C.2: Effect on Firm-Level Capital-to-Intangible Ratio

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We estimate the regression in three samples: (1) including all firms, (2) including only firms that are not balanced, and (3) including only firms that are balanced. Balanced firms are firms that have observations for all years. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

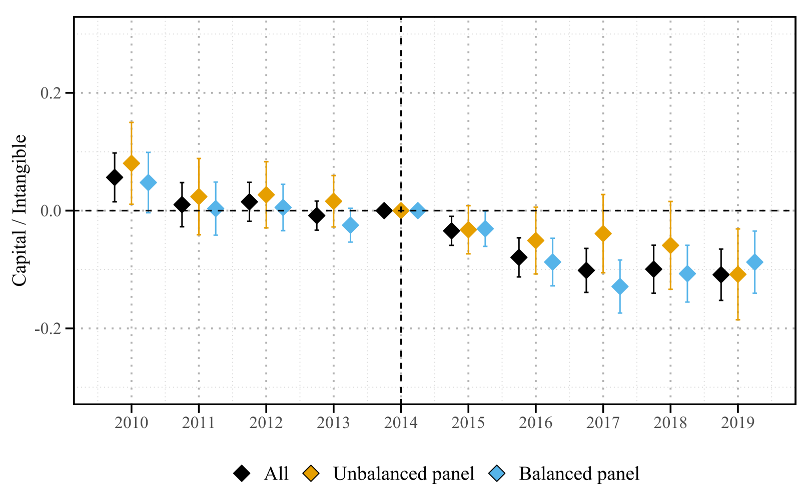


FIGURE C.3: Effect on Firm-Level Capital-to-Intangible Ratio

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We estimate the regression in three samples: (1) including all firms, (2) including only firms that did not have any positive profits before taxes between 2010 and 2014, and (3) including only firms that had at least one year of positive profits before taxes between 2010 and 2014. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

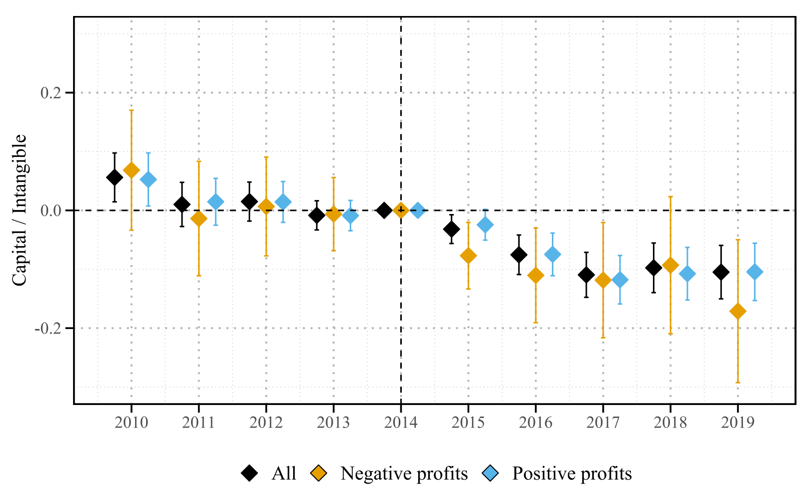


FIGURE C.4: Effect on Firm-Level Capital Stocks - Decomposition by Region

This Figure presents the results of estimating equation (6) on a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is either the logarithm of the stock of physical capital at the end of the period or the logarithm of the stock of intangible capital at the end of the period. We include firm and year fixed effects, as well as a vector of time-varying firm controls: logarithm of sales, the logarithm of total assets, the logarithm of the number of workers, leverage, the ratio of cash holdings to total assets, the ratio of EBITDA to total sales, the ratio of physical capital to total assets, and the logarithm of total sectoral sales. We present the average treatment effects over time, where we compare treated vs. control firms, using 2014 as the base year. Therefore, the coefficient for the year $2014 + m$ can be interpreted as the average treatment effect on the outcome variable between 2014 and $2014 + m$. We present results for three samples: (1) including all firms, (2) including only firms in regions where the investment subsidy was 10 percent, and (3) including only firms in regions where the investment subsidy was 25 percent. We cluster errors at the firm level and present both the coefficients and a 95 percent confidence interval.

(a) Physical Capital | (b) Intangible Capital

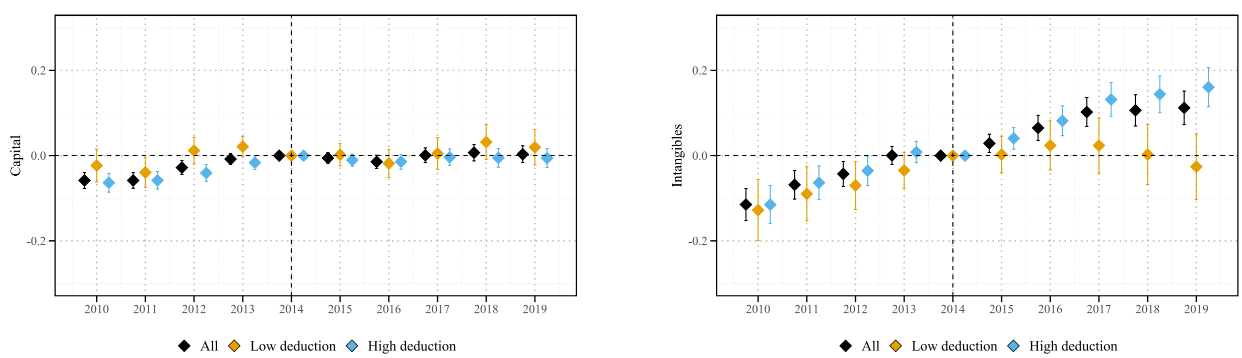
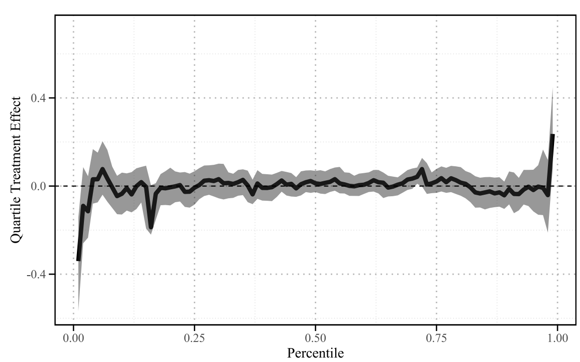


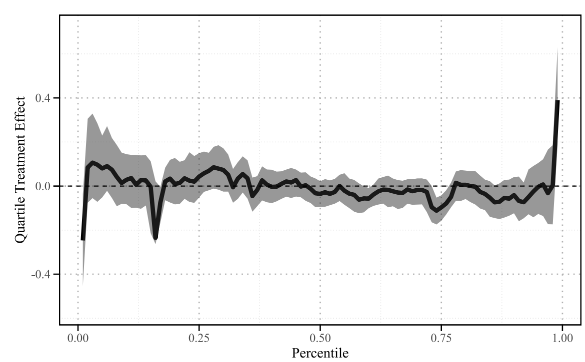
FIGURE C.5: Effect on Distribution of Capital-to-Intangible Ratio

This figure presents the results of a Change-in-Changes estimation using a sample of 115,529 firms with a total of 469,859 observations. The outcome variable is the logarithm of the ratio of physical capital at the end of the period (measured using PP&E) and intangible capital at the end of the period. For each percentile τ of the distribution of the outcome variable for treated firms in 2014 (the base year), we estimate the quantile treatment effect (QTE) between each year $t \geq 2015$ and the base year of 2014. We compute standard errors via a nonparametric block bootstrap that resamples firms (rather than individual observations) and present 95 percent confidence intervals.

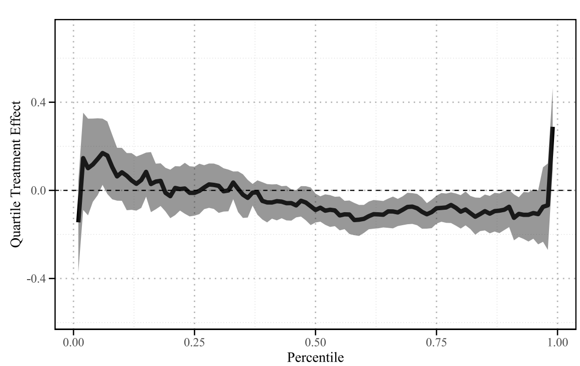
(a) 2015



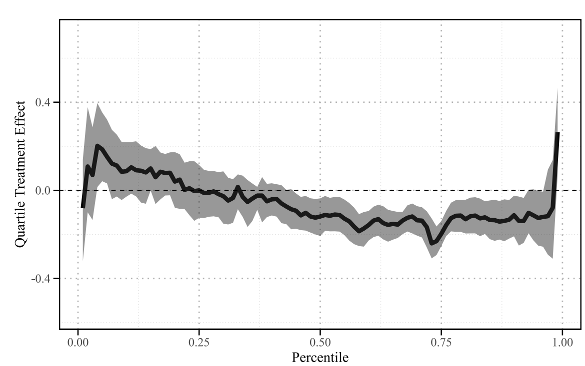
(b) 2016



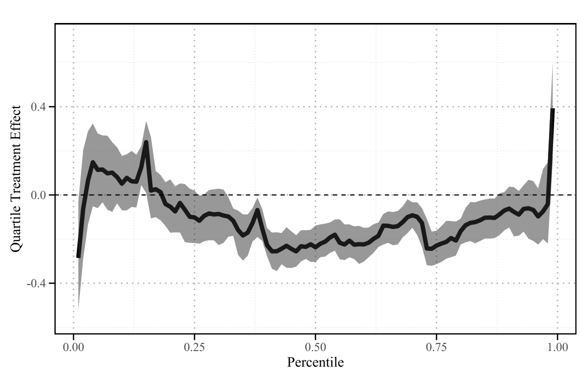
(c) 2017



(d) 2018



(e) 2019



D Constructing the Distribution of the Wedge

Let $X \sim F$ and let $U \equiv F(X)$ denote the percentile rank of X. By the probability integral transform, $U \sim \text{Unif}(0,1)$ under continuity of F; if F has atoms, a randomized rank $U = F(X^{-}) + V\,[F(X) - F(X^{-})]$ with $V \sim \text{Unif}(0,1)$ ensures the same result. We are interested in the distribution of

$$Y = g(U),$$

where $g : [0,1] \to \mathbb{R}$ denotes the wedge function estimated across percentiles.

D.1 Piecewise-linear representation of g

The function g is observed on a grid of percentiles $0 \leq p_1 < \cdots < p_N \leq 1$ with corresponding values $g_i = g(p_i)$. We sort by p, drop duplicates, and pad the endpoints so that $p_1 = 0$ and $p_N = 1$ if needed. Between consecutive grid points, we linearly interpolate:

$$g(p) = g_i + \frac{g_{i+1} - g_i}{p_{i+1} - p_i}\,(p - p_i).$$

For each segment, define the width $\Delta p_i = p_{i+1} - p_i$ and slope $\Delta g_i = g_{i+1} - g_i$.

D.2 Distribution induced by linear segments

Conditioning on the segment $[p_i, p_{i+1}]$, the probability that U falls in this interval is Δp_i. If $\Delta g_i \neq 0$, the linear map from p to y is invertible, and Y is uniform on the image interval

$$[y_{\min,i}, y_{\max,i}] = [\min(g_i, g_{i+1}), \max(g_i, g_{i+1})].$$

The conditional density on this interval is constant and equal to

$$f_{Y|i}(y) = \frac{\Delta p_i}{|\Delta g_i|}.$$

If $\Delta g_i = 0$, the entire probability mass Δp_i collapses to the point $y = g_i$, creating an atom.

Aggregating across segments yields the piecewise-constant density for the continuous part:

$$f_Y(y) = \sum_{i:\Delta g_i \neq 0} \frac{\Delta p_i}{|\Delta g_i|}\,\mathbf{1}\{y \in [y_{\min,i}, y_{\max,i}]\},$$

and point masses at the levels where g is flat:

$$\Pr(Y = c) = \sum_{i:\Delta g_i = 0,\, g_i = c} \Delta p_i.$$

The corresponding CDF is

$$F_Y(y) = \sum_{i:\Delta g_i \neq 0} \Delta p_i \frac{\big(y - y_{\min,i}\big)_+}{y_{\max,i} - y_{\min,i}} \wedge \Delta p_i + \sum_{i:\Delta g_i = 0} \Delta p_i \, \mathbf{1}\{y \geq g_i\},$$

where $a_+ = \max(a, 0)$ and $x \wedge b = \min(x, b)$.